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F. W. DYSON, Esq., M.A., LL.D., F.R.S., PRESIDENT, in the Chair.

Sarat Chandra Bhattacharya, M.A., F.C.S., F.R.Met.Soc., Professor, St. Columba's College, Hazaribagh, Bengal, India;
Rev. Martin Davidson, B.A., B.Sc., 56 Hudson's Road, Canning Town, E.;
Philip Fox, Director of the Dearborn Observatory, Evanston, Illinois, U.S.A.;
John Rudd Leeson, M.D. Edin., F.L.S., F.G.S., Clifden House, Twickenham, and Athenæum Club, S.W.; and
Hugh Wright, Librarian, Mitchell Library, Sydney, New South Wales, Australia,

were balloted for and duly elected Fellows of the Society.

The following candidate was proposed for election as a Fellow of the Society, the name of the proposer from personal knowledge being appended:—

Walter Bernard Kendall Francis, Chaplain and Naval Instructor, Royal Navy, Royal Naval Barracks, Shotley, Harwich (proposed by the Rev. D. Powell Richards).

Seventy presents were announced as having been received since the last meeting, including, amongst others:—

Francis Baily, Journal of a Tour in unsettled parts of North America, 1796–7, presented by Mr. William De Morgan.

Seven frames of photographs: spectroheliograms of faculæ and prominences, etc., taken at the Kodaikánal Observatory, presented by Mr. G. T. Walker; Photographs of Comets Brooks and Beljawsky (fourteen lantern slides), taken by H. Knox Shaw, presented by the Khedivial Observatory, Helwân.

Twenty-five Charts of the Astrographic Chart of the Heavens, presented by the Tacubaya Observatory.

Future Eclipse Observations.

(*Communicated by the Secretary, J.P.E.C.*)

The Joint Permanent Eclipse Committee has lately taken the subject of future eclipse observations into consideration, and is of opinion that work along the following lines is likely to prove of value and that opportunities of obtaining observations should be utilised.

These notes are published in order to assist any intending observers to make their plans.

(1) Direct photographs of the corona for record of form, in continuation of previous observations:
Also large scale photographs of the corona at widely separated stations to detect changes or movement, particularly of the "arches" in the inner corona.

(2) Photographs of the flash spectrum, particularly with a view of detecting differences, both as to relative intensities of lines and heights of strata, at different epochs in the sun-spot cycle. Wave-lengths are probably better determined apart from eclipses, as at Mount Wilson.

(3) Photographs of corona spectrum, particularly in ultra-violet, with an instrument of greater light grasp than used hitherto:
Also spectrum of the outer corona to study the occurrence of the Fraunhofer lines:
Also a study of the distribution of coronium in the corona:
Also a spectroscopic measurement of the rotation of the corona.

(4) Polariscopic measurements, *i.e.* distribution of polarisation in different parts of the corona, particularly "streamers" compared with "arches":
Also polarised spectra as a means of discriminating light due to scattering from that due to other possible causes.

(5) Measurement of the heat of the corona; but this is probably only possible in a long totality.

The determination of differential star places by photographic methods. First paper. By H. H. Turner, D.Sc., F.R.S., Savilian Professor.

1. In *Monthly Notices*, lxxi. p. 416, two methods were sketched by which differential places could be determined photographically; one by the use of a moving plate, and the other by the use of a cœlostat which could be set back to the same position at regular intervals, and could thus photograph on a fixed plate in a fixed telescope successive fields of stars of the same declination, and differing in R.A. by intervals of time measured on some form of chronograph, the particular form suggested being photographic. In preparing to make trial of this method, some difficulties were encountered, not due to the method itself, but simply to the small size of the only room at the University Observatory which was readily available at the moment: and in considering how to avoid these difficulties, a still simpler plan suggested itself which seemed well worth trial in the first instance; for if successful, it could be used by anyone possessing an ordinary photographic equatorial.

2. The chief modification is this, that the intervals between successive plates, instead of being determined by registering on any chronograph, are to be obtained from an overlapping plate or plates. Let us denote the first plate by AB, the second by CD, the third by EF and so on. Then we can take an overlapping plate BC which shall have half in common with AB and half in common with CD, and thus bridge the unknown gap between them. Suppose there are on the average n star images on each of the half-plates represented by the letters A, B, C, etc. Then if n is as large as the number of chronograph dots which would be conveniently used in the method as originally sketched, we get just as good a connection in this way as by the chronographic method. To strengthen the connection further, we may take *two* overlapping plates, dividing the interval into thirds. Thus let us represent successive plates by three letters ABC, DEF, etc. With the overlaps the whole series will be

ABC, BCD, CDE, DEF, etc.

Each star will appear on at least three plates (and sometimes on four if there is a margin of overlap in the original series), and we need not retain any distinction between the original series and the overlaps.

3. To give the method its best chance, we should use a fixed telescope as suggested in the paper above cited; the necessary movement during the exposures being provided either by a cœlostat or a moving plate. But it seemed well worth trying whether the ordinary equatorial would suffice, since this would involve no disturbance of ordinary observatory work. As a summary of what follows, it may be stated here that the results

are sufficiently promising to encourage us to carry out the complete survey of zone $+29^{\circ}$ by this method; but one or two circumstances have delayed the presentation of the results.

4. The first of these was literally a "screw loose" in the instrument. It was expected that the telescope would continue pointing to the same declination throughout, but on taking the first plate there was found to be a slow change, first in one direction for about $1''$ in 3 minutes, and then in the other at a gradually diminishing rate, approaching a final position asymptotically removed some $12''$ away from the first. The change was not in the plate, for there was no change of orientation, and it did not seem impossible that a slow change of this magnitude might be unavoidable, for we have practically nothing to guide us in such matters. [In taking a long exposure, for instance, the behaviour of the instrument would be mixed up with changes in refraction.] But in taking a second plate on May 24, special attention was paid to firm clamping, and yet the same thing happened. On making a thorough search it was found that a screw in the slow motion had worked loose and allowed of a little play, which was probably the seat of the trouble, but the measures on the plate taken since this was rectified are not yet sufficiently reduced to say definitely. A more vigorous search would have been made if the results had been vitiated by this trouble, but they were not. The look of them is spoiled a little, but in essentials they are just as good as if the telescope had not wandered, as may be seen below, and even if the trouble is not now removed (as is quite possible) the method is available nevertheless. But the investigation of the matter naturally caused a little delay.

5. The next trouble was this: In order to get as small a value as possible for error of orientation of the first plate, no *réseau* was impressed on the plate itself; but a *réseau* on another plate was placed film to film with it and adjusted so that the lines ran as nearly as possible in the right orientation when a little shellac was run in under the corners and the plates thus secured together. The effect on the error of orientation was successfully obtained, but the method brought troubles with it in the shape of a systematic error at the edges of the *réseau*. That the error is due in some way to this superposed *réseau* seems conclusively proved by the fact that it did not appear in the second plate, which was impressed in the ordinary way with the *réseau* before development. The matter is probably worth following up for its own sake, but for the present must be put aside.

6. A more serious cause of delay is the difficulty of testing the value of the results obtained. These are revised places of the stars used for finding the constants of the plates, and of many others in addition. It is easy to substitute these revised places and show that the scale values and orientations deduced from the x and y measures are now in much better accord, and this has been done for a sufficient number of test cases. But there remain

the coordinates of the plate centres. Probably the best plan of demonstrating the value of the method will be to repeat the same region twice and show the accordance of the results. But as yet this has not been done, owing to the necessity of proceeding tentatively. The deduced places have, however, been compared with the Cambridge meridian observations and with the Oxford "Ledgers" of corrections to these; and, as an incidental outcome, the comparison led to the detection of a new fact about the "magnitude equation" of the Cambridge meridian observations, viz. that it apparently changed character about 1879. This result has been confirmed by the discussion of the first 3000 stars (0^h–6^h), and the investigation will be carried through the whole Catalogue. The results up to the present are as follows:—

Cambridge Magnitude Equation in R.A.

B = Before 1879·0. A = After 1879·0.

(The unit is 0″·3.)

Mag.	B.	A.	A–B.	Mag.	B.	A.	A–B.
7·9–8·1	+2·6	+1·7	−0·9	9·1	−0·4	−2·6	−2·2
8·2–8·4	+2·4	+1·4	−1·0	9·2	−1·7	−3·4	−1·7
8·5–8·7	+1·6	+0·8	−0·8	9·3	−0·9	−3·5	−2·6
8·8–8·9	+1·4	+0·2	−1·2	9·4	−2·5	−3·5	−1·0
9·0	+0·4	−0·4	−0·8	9·5	−2·5	−4·7	−2·2
		Mean	− ·9			Mean	−1·9

(For bright stars the two periods are fairly accordant.) This difference does not directly concern the present investigation, but it would not have been discovered if the places obtained had not been good enough to furnish the clue.

Method of taking the plate.

7. Having thus generally indicated the stage which the investigation has reached, we proceed to details, and first as regards taking the plate. The following description of a particular case will suffice for all.

On 1911 April 3 plate No. 2863 was put into the Astrographic Equatorial, pointed to $8^h\ 46^m + 29^\circ$ (one of the normal plate centres of the Oxford zones) near the meridian. After the clock had been seen to have a good rate, the guiding telescope was no longer used; but the position of a handle for winding the sector, which makes one revolution in 90 seconds, was noted at a time T. The clock was allowed to drive for $1\frac{1}{2}$ minutes longer and then an exposure of one minute was given. The clock was then disconnected, and during the next half minute the sector was wound back to its position at time T. These operations were repeated every 3 minutes until clouds came up about an hour and a half later. The order of events is thus—

Time.	Position of Sector.	Exposure.
	m	
T + 0	First	...
T + 1½	Second	Begins
T + 2½	Third	Ends
T + 3	First	...
T + 4½	Second	Begins
T + 5½	Third	Ends
T + 6	First	...
T + 7½	Second	Begins
T + 8½	Third	Ends

and so on. The plate had in all 28 exposures of 1 minute each, to plate centres ranging from R.A. $8^h\ 46^m$ to R.A. $10^h\ 7^m$, at 3 minute intervals, and recorded the positions of 181 stars of the Cambridge Catalogue (as well as many others) three or four times over.

8. *Method of Reduction.*—The reduction of such a plate is extremely simple, and need not be very laborious if proper tables are made. It need not be explained here how to correct the measures *approximately* for scale value, orientation, and plate centre; rough places of one or two stars will give all that is needed to convert the measures into *approximate* standard coordinates, so that the residual systematic errors shall not exceed ·005 (or 1″·5) in any part of the plate. The final correction will be made later.

9. Now convert these approximate standard coordinates into X and Y in the Oxford notation: where

$$X = (\alpha^s - A^s)/20 \quad \text{and} \quad Y = (\delta'' - D'')/300$$

(α, δ) being the R.A. and Decl. of any star image, and (A, D) those of the plate centre. This conversion is not a very serious matter when tables have been formed for each plate centre, as in the Oxford volumes.

We must now distinguish the images belonging to the successive exposures. Call them $(X_1\ Y_1)$, $(X_2\ Y_2)$, etc. Form all the differences between X_n and X_{n+1} for such stars as appear both in the n^{th} and $(n+1)^{th}$ exposures. This gives the interval between these exposures, and so for every pair. Adding the links, we get the interval between the first exposure and the last. Now take a known star in each of these exposures (or several if preferred), and we have the interval between these two as measured on the plate in terms of our adopted scale to compare with the interval in the Catalogue, which gives us a good scale value. In the present case this interval is about 5000^s, so that an error of $0^s{\cdot}1$ in either element of comparison would give us the scale value correct to ·00002. We ought to be able to do at least as well as this, even independently of the information given by the general mass of the stars.

10. Turning now to the values of $Y_n - Y_{n+1}$, if these remain constant we may attribute them to the orientation, which displaces a star by a constant amount in a constant interval. As will be seen below, these differences did *not* remain constant, and part

of them must therefore be attributed to a shift of plate centre. The shift was in one sense for a few minutes, and then in the contrary for the rest of the time; *something* was slowly changing for an hour and a half. A second plate taken on May 24 showed much the same behaviour. It was found later that a screw in the slow motion had worked loose, and we need not follow the matter further until it is seen whether a plate taken since the screw was put right still shows the changes.

11. Meantime there is no reason why these changes should vitiate the determination of differential places. They merely put the y measures on the same footing as the x measures, in which an unknown interval separates the exposures. Excepting only that we cannot now determine the orientation without reference to known stars, we lose nothing of importance by these small shifts.

One question, however, does arise. If the plate centre shifts, does the orientation change? The answer is apparently in the negative. If there is a change of orientation, the differences in X for the top of the plate should not agree with those at the bottom; yet they do, as will be seen below.

12. We now proceed to give the detailed evidence in support of the above conclusions:—

TABLE I.

Differences between Exposures in X.

(The unit is $20^{sec.}$)

R.A. h m	Difference.	No. of Stars.	R.A. h m	Difference.	No. of Stars.
8 46				(8·9936)	(11)
	9·1419	11	9 28		
8 49				9·0011	18
	8·6863	9	9 31		
8 52				9·0030	22
	8·9985	14	9 34		
8 55				8·9934	13
	8·9995	20	9 37		
8 58				9·0008	11
	9·0025	18	9 40		
9 1				8·9974	9
	8·9954	19	9 43		
9 4				8·9993	11
	8·9950	13	9 46		
9 7				8·9950	14
	9·0046	10	9 49		
9 10				9·0005	18
	8·9961	16	9 52		
9 13				8·9956	16
	9·0005	14	9 55		
9 16				9·0070	10
	8·9986	14	9 58		
9 19				8·9954	13
	8·9894	15	10 1		
9 22				8·9959	15
	9·0004	11	10 4		
9 25				8·9990	16
	8·9936	11	10 7		
				242·7857	

Little explanation is needed of Table I. The unit adopted is 20^{sec} (which represents a *réseau* interval at the Equator), and it will be seen that the differences are all close to 9 units, or 3^{min}; except in the first two cases, where the method of setting had not been quite grasped.

13. Let us now determine the correct scale value, by taking a couple of stars on the first plate and on the last, and comparing the measured interval with that on the sky.

Star's No. in Camb.	R.A. 1900·0.	Δα.	Precession, etc.	R.A. at date.	X_c.	X_o.	$X_o - X_c$.
	h m s	s	s	h m s			
4706	8 44 34·98	0·00	+40·73	8 45 15·71	−2·2145	−4·5242	−2·3097
4712	8 45 45·27	+0·07	+40·69	8 46 26·03	+1·3015	−1·0027	−2·3042
				Mean	−0·4565	−2·7635	−2·3070
5251	10 7 1·99	+0·03	+38·99	10 7 41·01	+2·0505	+0·0923	−1·9582
5254	10 7 27·20	+0·10	+39·00	10 8 6·30	+3·3150	+1·3599	−1·9551
				Mean	+2·6827	+0·7261	−1·9566

The correction Δα is derived from the Oxford ledgers of corrections, and stars have been chosen which have obviously very small P.Ms. The next column gives star correction and precession to date of plate: for we deal with *apparent places at date* just as in meridian work. The X_c is then the calculated X. The whole Catalogue difference between the pairs of stars is thus (in the units adopted for X),

$$243 + 2{\cdot}6827 + 0{\cdot}4565 = 246{\cdot}1392.$$

The measured difference in adopted units is

$$242{\cdot}7857 + 0{\cdot}7261 + 2{\cdot}7635 = 246{\cdot}2753,$$

and hence the measures must be *reduced*. The reducing factor is

$$1 - {\cdot}1361/246{\cdot}3 = 1 - {\cdot}00055.$$

14. Now this is a large factor, and should have been detected from the measures and corrected before they were converted into X and Y. Indeed at first it was thought that the preliminary correction which had been made (viz. 1 + ·00033) had been applied with the wrong sign. This was not so; but there was an oversight of a different kind.

The error of centre in η was +0·64 approximately in *réseau* intervals, and this should have been subtracted from all the measures of η before using them to find X from the tables. It is readily seen, either by inspection of the tables given in the Oxford volumes, or by consideration of the theoretical formula, that *a small error in the η of the plate centre produces no sensible error in Y, but an erroneous scale value in X.*

Take the check from the tables: on p. xl of the Oxford volume iii. (29° zone) the formula for X is

$$X = \xi + \tfrac{1}{8}\xi + \tfrac{1}{160}\xi + \text{following table.}$$

If we make an error in η nothing is altered but the table. Now when

$$\xi = 10{\cdot}0 \text{ and } \eta = -13{\cdot}00, \text{ table gives } {\cdot}0014.$$
$$\xi = 10{\cdot}0 \text{ and } \eta = -12{\cdot}36, \quad \text{,,} \quad {\cdot}0071.$$

The difference of ·0057 in 10 or ·00057 in 1 is almost exactly what we require. To show that it is practically a simple change of scale value, take values of η at the top of the plate instead of the bottom on p. xlv. When

$$X = 10{\cdot}0 \text{ and } \eta = +12{\cdot}36, \text{ table } = {\cdot}0190.$$
$$X = 10{\cdot}0 \text{ and } \eta = +13{\cdot}00, \text{ table } = {\cdot}0250.$$

The difference is ·00060 instead of ·00057. The assumption is thus not exact, and to avoid making any assumption it is better to avoid the oversight. But small errors may happen by accident, and then they can be corrected in this way.

The sign of the correction is easily checked if we remember the way in which the meridians converge to the pole. If our measures put the stars too near the pole, we shall unduly expand them and the X's will be too large. This is what has happened in the present case.

15. To sum up, we may regard the adopted scale value as correct in Y; it would have been correct in X but for the oversight of omitting to subtract the correction 0·64 to the η of the plate centre; but in consequence of this the X's require multiplying by (1 − ·00058).

But the application of this correction is very easy. We need not trouble to apply it to the individual measures, but only to their mean. Now, since the various images of a star range across the plate, its mean X will always be small. The extreme case is when one image appears near one edge, say at $X = -12$, and then at $X = -3$ and $X = +6$, the fourth image at $X = +15$ just not appearing. The mean for the three which do appear is thus $X = -3$, and the correction is $+{\cdot}00058 \times 3 = +{\cdot}0017 = +0''{\cdot}5$, which is the maximum possible.

16. We proceed to consider the residual differences, and as a check on systematic errors let us arrange them in order of X and of Y. It is unnecessary to give the individual differences, which naturally show deviations from the means tabulated below. At present we are concerned with possible systematic errors.

TABLE II.

$\Delta X = (X_n - X_{n+1})$ *arranged according to* X.

(The unit is 0·0001 or 0″·03 approximately.)

X.	ΔX.	No. of Stars.	Corr. for *Réseau.*	X.	ΔX.	No. of Stars.	Corr. for *Réseau.*
−14·5 to −5·5	+38	15	− 6	−3·5 to + 5·5	+ 6	23	− 1
−13·5 to −4·5	+ 3	11	− 4	−2·5 to + 6·5	0	20	0
−12·5 to −3·5	+14	21	− 4	−1·5 to + 7·5	+ 4	15	0
−11·5 to −2·5	+10	21	− 4	−0·5 to + 8·5	− 4	17	0
−10·5 to −1·5	+ 8	15	− 4	+0·5 to + 9·5	− 4	27	+ 1
− 9·5 to −0·5	+ 7	15	− 4	+1·5 to +10·5	− 2	26	+ 3
− 8·5 to +0·5	0	25	− 2	+2·5 to +11·5	−12	16	+ 3
− 7·5 to +1·5	− 3	29	− 4	+3·5 to +12·5	−14	14	+ 5
− 6·5 to +2·5	− 6	15	− 2	+4·5 to +13·5	−12	14	+ 5
− 5·5 to +3·5	− 4	16	− 1	+5·5 to +14·5	−31	12	+ 7
− 4·5 to +4·5	+ 2	13	− 1				

17. Now there is here a systematic error of a curious kind. It is roughly proportional to $-X$; and since we are dealing with differences, it implies a term in the original X proportional to $-X^2$; let us put

$$\Delta X = -kX^2.$$

Then

$$\Delta(X+9) - \Delta X = -k(X+9)^2 + kX^2$$
$$= -18kX + \text{const.}$$

Taking the mean of the three extreme values at each end of the series, we get a change of the error from +18 to −19 units of ·0001 for an interval of 18 in X: *i.e.*,

$$-18k \times 18 = -\cdot0037$$
$$\therefore \quad k = -\cdot000011.$$

A term in X^2 naturally suggests an error of tilt; but if so, the term $-kX^2$ in X must be accompanied by a term $-k^1XY$ in Y, where $k^1 = k \sec D$. [If we use ξ and η, k^1 would be the same as k.] Hence we can test the hypothesis that the plate was seriously tilted by examining the Y differences. We have

$$\Delta Y = -k^1XY,$$

and the Y *differences* between images separated by 9 units in X give us the quantity

$$-k^1(X+9)Y + k^1XY = -9k^1Y,$$

i.e. they ought to show a progressive change when arranged according to values of Y. But they do not, as may be seen from Table III., which should properly come in the discussion of the Y differences, but may be given here by anticipation.

TABLE III.

$\Delta Y = Y_n - Y_{n+1}$ *arranged according to* Y.

(The unit is 0″·03.)

Value of Y.	Δ Y.	No. of Stars.	Value of Y.	Δ Y.	No. of Stars.
−12	+4	17	+ 2	−1	33
−10	+2	13	+ 4	0	34
− 8	+1	43	+ 6	+3	31
− 6	−3	36	+ 8	+3	23
− 4	0	30	+10	+2	16
− 2	+1	35	+12	−8	38
0	−2	32			

18. The first value of Δ Y is positive and the last is negative, but if we take the mean of the first four groups and the mean of the last four we get a coefficient which is practically insensible. The tilt can only be small, and we must look for some other origin for the systematic differences in Table II.

19. The *réseau* was suspected, and on looking up the table of its errors furnished by the makers (Messrs. Gautier), it was found that they were in the right direction, but much too small. They are shown in the third column of Table II. as calculated from the figures given by the makers MM. Gautier.

The error in question, which is a systematic increase in the intervals with X, can readily be detected by placing two copies of the *réseau* film to film, so that the small X's of the one fall on the large X's of the other, and then measuring the distances between the lines across the plate. This was done, and it was found that the errors were of the same general nature as are shown in Table II., but smaller. [It is by this time well known that copies do not exactly reproduce the errors of the *réseau*.]

20. To test this curious anomaly further another plate was taken, and the *réseau* impressed in the usual way. The differences, when corrected for the errors of the *réseau*, show no sensible run in X.

21. The most probable supposition seems to be that the residual discordances on the plate now under consideration are due in some way to the fact that it was measured with a *réseau* on another plate, bound up with it. We cannot be sure of this without further experiments, but the relative curvature of the two plates (even though both are of plate glass) might explain the errors, which are, after all, not very large. For the sake of illustration, we will write down a set of errors in X which might explain the errors in the differences (X + 9) − X. They are assumed to be symmetrical about the plate centre; and it will be seen that the middle portion of the plate from −7·5 to +7·5 is regarded as practically free from error. The suggested corrections include those due to the *réseau*. It will be seen that the errors of the differences are greater than those of the single measures, owing to the errors going opposite ways.

TABLE IV.

Suggested Errors in Measures of X.

(The unit is 0·0001 or 0″·03.)

X.	Error in X.	Error in (X+9).	Difference.	Observed.	O−C.
−14·5	−18	+ 9	+27	+38	+11
−13·5	−10	+ 9	+19	+ 3	−16
−12·5	− 8	+ 7	+15	+14	− 1
−11·5	− 5	+ 5	+10	+10	0
−10·5	− 3	+ 5	+ 8	+ 8	0
− 9·5	− 1	+ 5	+ 6	+ 7	+ 1
− 8·5	+ 2	+ 5	+ 3	0	− 3
− 7·5	+ 5	+ 5	0	− 3	− 3
− 6·5	+ 8	+ 5	− 3	− 6	− 3
− 5·5	+ 9	+ 7	− 2	− 4	− 2
− 4·5	+ 9	+ 9	0	+ 2	+ 2
− 3·5	+ 7	+ 9	+ 2	+ 6	+ 4
− 2·5	+ 5	+ 8	+ 3	0	− 3
− 1·5	+ 5	+ 5	0	+ 4	+ 4
− 0·5	+ 5	+ 2	− 3	− 4	− 1
+ 0·5	+ 5	− 1	− 6	− 4	+ 2
+ 1·5	+ 5	− 3	− 8	− 2	+ 6
+ 2·5	+ 5	− 5	−10	−12	− 2
+ 3·5	+ 7	− 8	−15	−14	+ 1
+ 4·5	+ 9	−10	−19	−12	+ 7
+ 5·5	+ 9	−18	−27	−31	− 4

22. Now, though these assumed errors may not be exact, they are probably not far from the truth. Suppose now we consider the effect on the mean of the sets of star images. If an image occurs at −14·5 it will also occur at −5·5, +3·5, and 12·5: if at −13·5 it will also occur at −4·5, +4·5, and 13·5: if at −12·5, it occurs also at −3·5, +5·5, and +12·5. In all other cases there are only three images. Let us write down the errors of the various exposures—they will repeat after we pass the middle of the plate, and we need not repeat them here.

TABLE V.

				Mean.	Adopted Mean.
(−18)	+9	+7	− 8	−3	+3
[−10]	+9	+9	[−10]	0	+3
− 8	+7	+9	(−18)	−3	+3
− 5	+5	+8		+3	+3
− 3	+5	+5		+2	+2
− 1	+5	+2		+2	+2

23. If, then, we omit the extreme outside lines (in round brackets) and give half weight to the next inside them (in square brackets) we get the adopted means in the last column, which give the plate sensibly free from error. We cannot, of course, extend this procedure to the Y coordinate without losing stars altogether, but comparison with overlapping plates will tell us whether there is any similar error in Y. But it may reasonably be hoped that this case is exceptional, and due to the use of a detached *réseau* as already mentioned.

24. In Table V. the X differences are arranged in order of Y, and it will be seen that there is nothing systematic which need concern us. But this negative result is nevertheless of importance. If there had been a change of orientation, it would have diminished the differences at the top of the plate and increased them at the bottom, or *vice versa*. Hence the table is evidence against a *persistent* change at any rate. And the detailed results are against a change first one way and then the other.

TABLE VI.

X *differences arranged according to* Y.

(The unit is ·0001 or 0″·03.)

Y.	Δ X.	No of Stars.	Y.	Δ X.	No. of Stars.
−13	−5	8	+ 1	+ 5	15
−12	−1	9	+ 2	+ 4	18
−11	+3	5	+ 3	+ 5	19
−10	−8	8	+ 4	+ 3	15
− 9	+2	13	+ 5	− 3	16
− 8	+1	30	+ 6	− 3	15
− 7	+2	23	+ 7	−11	7
− 6	+3	13	+ 8	− 2	16
− 5	+8	8	+ 9	− 5	12
− 4	0	22	+10	0	4
− 3	−1	24	+11	+ 1	14
− 2	6	11	+12	− 2	22
− 1	+4	14	+13	− 6	2
0	0	18			

25. We are, therefore, in a position to deduce *differential* X coordinates of the stars if we know the orientation. This is to be discussed in connection with Y, and for the present we will merely consider it determined empirically. Applying it and the scale value as already explained, and the coordinates of centre for each exposure from Table I. (taking our starting-point from the four stars 4706, 4712, 5251, and 5254), the X coordinates of all the other stars were found.

26. Now it is of little importance for our present purpose

to give these coordinates themselves. What we are concerned with is their comparison with others. For instance, how do they compare with the places of the Cambridge Catalogue? These are based on observations made in the years 1874–1893; but, in the ordinary course of the work of finding constants for the Oxford plates, they were compared with the photographic measures (1893–1909) at Oxford, and the differences entered in Ledgers where the results for the same star from different plates are collected. Since the Ledgers represent an undoubted improvement on the Cambridge Catalogue (including, as they do, the correction of a conspicuous magnitude equation, as well as partial corrections for proper motion), the first comparison was made with the ledgers, and to test a possible change in orientation the results were grouped in order of Y. The means were as follows:—

TABLE VII.

Mean Corrections from Plate 2863 to the Oxford Ledgers.

(The unit is 0·0001 or 0″·03 approx.)

Mean Y.	Mean Correction.			No. of Stars.	Leiden Correction.	Final.
	1st Half.	2nd Half.	Whole.			
+11	+ 3	+ 6	+ 4	19	− 4	0
+ 7·5	+16	− 4	+ 7	22	0	+ 7
+ 4	− 2	0	− 1	23	+ 2	+ 1
+ 1	−21	−13	−17	24	+ 6	−11
− 1	+ 2	+ 7	+ 5	21	...	+ 5
− 4	+ 8	+ 5	+ 7	24	...	+ 7
− 7	+ 2	−16	− 6	27	...	− 6
−10	+ 5	− 5	− 1	17	...	− 1

27. The material was divided into two groups (in the 2nd and 3rd columns) as a test whether the orientation had changed. As there seems to be no sensible change, the groups are combined according to number of stars in the 4th column. The abrupt change between Y = + 1 and Y = − 1 naturally attracts attention. It is unlikely to be in the plate 2863, but may easily arise from the division of the plates represented in the ledgers.

28. A partial explanation of it is indicated in the 6th column, based on special calculations already made, and it arises as follows:—

For the zone + 29° all the stars are contained in the Cambridge Catalogue, which was one of the reasons for selecting it. But zone + 30° contains Cambridge stars in the lower half and Leiden stars in the upper. Now there is a systematic difference between the two catalogues, of which it was not possible to take account when the plates were commenced, and which was therefore left for treatment in the collected results. But the difference naturally affects the ledgers for stars between + 29° and + 30°. Suppose, for simplicity, we determined the constants for a plate with centre

+ 30° from two stars only, L a Leiden star in the upper half, and C a Cambridge star in the lower, and suppose that in the sky these have the same R.A. Owing to the systematic difference mentioned, C would be recorded with a greater R.A., and the deduced orientation would be erroneous. This would displace stars at the top and bottom of the plate, *i.e.* stars near + 31° and + 29°. The former would not concern us, but the latter displacement would introduce an error into the ledgers just above + 29° which would not exist just below it. A definite calculation of the effect is shown in Table VII., column 6, and it will be seen that the systematic change, though reduced, is not removed. The nature of the change should be remarked: it is a change in X as we proceed in the direction of Y, and thus could not be produced by a faulty *réseau* line: this would need a considerable and abrupt change in all the vertical lines.

29. If the change comes in through the breaking up of the Astrographic plates into zones, it ought not to exist in the original meridian observations (unless these also were made in zones). To test this a comparison was made, with the following results:—

TABLE VIII.

Corrections from Plate 2863 to the Cambridge places.

(The unit is 0·0001 or 0″·03 approximately.)

Mean Y	+11	+7·5	+ 4	+ 1	− 1	− 4	− 7	−10
O − C.	−18	+ 10	−11	−24	+ 8	+15	+ 9	− 6.

The accidental errors are manifestly larger, and the abrupt change between + 1 and − 1, now emphasized, may be merely accidental.

30. But the fact that it was emphasized suggested still further investigation, and a rather curious point was brought to light. Attention was drawn to the fact that the residuals seemed to vary with the mean date shown in the Cambridge Catalogue, and they were accordingly arranged, as a trial, according to this date. The results were as follows:—

TABLE IX.

(The unit of the mean resid. is approx. − 0ˢ·0020.)

Year.	1874,	1875,	1876,	1877,	1878,	1879,	1880-3,	1884-5,	1886-90,	1893-4.
No. of Stars.	13	28	40	14	20	9	26	13	13	5
Mean Resid.	− 1	+ 6	+ 3	+25	+ 1	−33	−42	−13	−16	−56

31. There is a conspicuous change about 1878, but this may be due to the selection of stars. It is known that there is a large "magnitude equation" in the Cambridge Observations; and if all the stars observed after 1878 were faint, the change might be explained. Accordingly, the stars were divided into two groups

before 1879 and after 1879, and the magnitudes were then taken into account :—

TABLE X.

Mag.	Before 1879·0.		After 1879·0.	
	No. of Stars.	Resid.	No. of Stars.	Resid.
<8·0	21	+34	7	+ 5
8·1 to 8·7	29	+11	10	+16
8·8 to 8·9	14	− 5	8	−58
9·0	31	− 4	10	−39
9·1 to 9·5	17	−18	21	−55
<8·7	50	+21	17	+11
8·8 to 9·5	62	− 8	39	−51

The last two lines summarize those above them, and indicate that the change is mainly in the faint stars. To test the reality of this change a totally different set of stars were examined, as shown in § 6. There seems to be no doubt of the change at Cambridge, and a letter was sent to Sir Robert Ball drawing his attention to it. It will also be considered in the discussion of the remainder of the Ledgers which is now in hand.

32. Nevertheless it did not seem that this change could explain the whole of the above anomaly directly. All that can be said is that until the whole history of this magnitude equation at Cambridge is unravelled, the places must be treated as subject to large accidental errors. There are various indications that we are not at the end of the investigation of personal equation in visual observations : even the moving wire method seems to have its peculiarities near the zenith. One of the chief reasons for urging the use of photographic observations is that we may hope to get rid of this troublesome and elusive error. As an illustration we may give here the mean differences between the places deduced from plate 2863 and the Oxford Ledgers (both photographic determinations), including, of course, 10 years' proper motion :—

TABLE XI.

X *of Plate* 2863 *compared with Oxford Ledgers.*

(The unit is 0·0001, or 0″·03, or 0^{s}·002.)

Mag.	6·0	7·0	7·8	8·2	8·7	9·0	9·3
No. of Stars	3	11	8	23	51	40	40
Diff.	0	−5	+1	+3	−2	+1	0

Of course this comparison is between plates taken with the same instrument; but the method of taking was different, and there was 10 years' interval. It is known that magnitude equation may come in through some defect in the lens; but if a watch is kept on this source of error, there is hope of avoiding many of the troubles to which eye observations are subject.

33. We now come to the Y residuals, but there is no necessity to give here the examination of them at the same length. We first form the mean differences between pairs of exposures as was done for X in Table I. They are shown in Table XII.

TABLE XII.

Differences between Exposures in Y.

(The unit is 0·0001 or 0″·03 nearly.)

Exp. h m	No. of Stars.	Diff. Y.	Corr. for Orientation.	Adopted.
8 46				
	11	−43	−30	
49				−30
	9	− 2	+11	
52				−19
	14	+18	+31	
55				+12
	20	+27	+40	
58				+52
	18	+24	+37	
9 1				+89
	19	+19	+32	
4				+121
	13	+23	+36	
7				+157
	10	+ 8	+21	
10				+178
	16	+ 6	+19	
13				+197
	14	+ 3	+16	
16				+213
	14	− 5	+ 8	
19				+221
	15	0	+13	
22				+234
	11	− 5	+ 8	
25				+242
	11	0	+13	
28				+255

Exp. h m	No. of Stars.	Diff. Y.	Corr. for Orientation.	Adopted.
9 28				
	18	+ 5	+18	
31				+273
	22	− 9	+ 4	
34				+277
	13	− 5	+ 8	
37				+285
	11	−14	− 1	
40				+284
	9	− 8	+ 5	
43				+289
	11	−15	− 2	
46				+287
	14	− 3	+10	
49				+297
	18	− 1	+12	
52				+309
	16	− 5	+ 8	
55				+317
	10	− 7	+ 6	
58				+323
	13	− 2	+11	
10 1				+334
	15	−15	− 2	
4				+332
	16	−15	− 2	
7				+330
	Sum	−19		

34. These differences are taken in the sense $Y_1 - Y_2$, *i.e.* the Y of the later exposure is subtracted from the earlier. They are partly due to orientation of the plate and partly to changes in plate centre. We may again remark on the evidence against change in orientation of the plate. If, for instance, the first result −43 were due to a change of orientation between the first and second exposures, then it would show in the X measures, the differences for upper and lower halves of the plate being different. But the mean of three X differences at

$$Y = +7{\cdot}1 \text{ is } +1436$$

and at

$$Y = -7{\cdot}0 \text{ is } +1437$$

whereas these should differ by 75 if the 43 were a change of orientation.

35. To find the total change in plate centre we have recourse to the same pairs of stars as before :—

Y 1900·0.		ΔY.	Precess., etc.	Y_c.	Y_o.	$Y_o - Y_c$.	X.
C 4706 (+29°)	−6·1257	−·0020	−·4639	−6·5916	−5·9230	+0·6686	−4·5
C 4712 (+29°)	−7·2537	+·0010	−·4669	−7·7196	−7·0490	+0·6706	−1·0
			Mean	−7·1556	−6·4860	+0·6696	
C 5251 (+29°)	−3·1690	0000	−·6409	−3·8099	−3·1740	+0·6359	+0·1
C 5254 (+29°)	−2·7057	0000	−·6421	−3·3478	−2·7106	+0·6372	+1·4
			Mean	−3·5788	−2·9423	+0·6365	

36. From these figures it is clear that at the last exposure the star images were +0·0331 lower on the plate than at the first. If we assume that the orientation of the plate remained constant and equal to v per interval of 20^{sec} (or $9v$ for 9 intervals which separates the exposures), then the images of the exposure $(n+1)$ will be higher on the plate than those of exposure n by $9v$ due to orientation, plus any change of plate centre. Hence, adding all the differences together,

$$27 \times 9v - ·0331 = +·0019 \text{ (from Table VI.)}$$
$$9v = +·0350/27 = +·0013$$
$$v = +·00014$$

37. The separate changes of centre are thus obtained by adding the quantity +·0013 to each of the Y differences, as shown in the fourth column of Table VI. It may be remarked that it does not matter whether they are due to changes in the instrument itself or to refraction, though only a minute portion of them can be due to the latter, since the total refraction is only 30″ and the barometer only changed from 30·03 to 30·05 during the whole series, and the thermometer from 36°·5 to 34°·7.

In the last column of Table XII. is given the cumulative correction to plate centre to reduce all the exposures to the first as standard.

38. The differences $Y_n - Y_{n+1}$ arranged according to Y have been given in Table III. and are satisfactorily small, so that they need not be further considered.

39. In Table XIII. are shown the same differences arranged according to X, and here we come on a small error of a rather puzzling kind, varying approximately with X, and hence denoting a term in X^2 in the Y coordinate. It is apparently not due to the superimposed *réseau*, for it exists also (and is slightly greater if anything) on plate 2868, which had a *réseau* impressed in the ordinary way. The same correction has been adopted for both plates, and is shown in the third column. (It is even a little

excessive perhaps, but it is better not to minimise the error.) If the error in Y be denoted by kX^2 then

$$k(X+9)^2 - kX^2 = \cdot00006X + \text{const.}$$

$$\text{or } k = \cdot00000333$$

The value of kX^2 at the edges of the plate ($X = -15$ say) is thus ·0007 in *réseau* intervals, less than the adopted unit of measurement. It is noteworthy that an error of this order should

TABLE XIII.

$Y_n - Y_{n+1}$ *arranged according to* X.

(The unit is 0″·03.)

X.	Plate 2863. $Y_{n+1} - Y_n$.	·00006.	Corr.	No. of Stars.	Plate 2868. $Y_{n+1} - Y_n$.	Corr.	No. of Stars.	Mean both Plates.
−14·5 to − 5·5	+ 8	− 6	+ 2	15	+17	+11	9	+ 6
−13·5 to − 4·5	0	− 5	− 5	11	+ 2	− 3	7	− 4
−12·5 to − 3·5	+12	− 5	+ 7	21	+10	+ 5	22	+ 6
−11·5 to − 2·5	+ 2	− 4	− 2	21	− 1	− 5	21	− 4
−10·5 to − 1·5	+ 5	− 4	+ 1	15	+ 3	− 1	10	0
− 9·5 to − 0·5	− 1	− 3	− 4	15	+ 4	+ 1	12	− 2
− 8·5 to + 0·5	+ 2	− 2	0	25	+ 2	0	24	0
− 7·5 to + 1·5	− 2	− 2	− 4	29	+ 6	+ 4	20	0
− 6·5 to + 2·5	− 3	− 1	− 4	15	+ 5	+ 4	16	0
− 5·5 to + 3·5	− 7	− 1	− 8	16	− 2	− 3	12	− 6
− 4·5 to + 4·5	− 5	0	− 5	13	− 2	− 2	15	− 3
− 3·5 to + 5·5	− 2	+ 1	− 1	23	− 3	− 2	18	− 1
− 2·5 to + 6·5	− 4	+ 1	− 3	20	+ 3	+ 4	21	0
− 1·5 to + 7·5	+ 2	+ 2	+ 4	15	+ 7	+ 9	10	+ 6
− 0·5 to + 8·5	0	+ 2	+ 2	17	− 8	− 6	11	− 2
+ 0·5 to + 9·5	− 3	+ 3	0	27	+ 1	+ 4	19	+ 2
+ 1·5 to +10·5	− 2	+ 4	+ 2	26	− 1	+ 3	22	+ 2
+ 2·5 to +11·5	− 1	+ 4	+ 3	16	− 3	+ 1	14	+ 2
+ 3·5 to +12·5	− 3	+ 5	+ 2	14	−11	− 6	15	− 2
+ 4·5 to +13·5	+ 3	+ 5	+ 8	14	+ 2	+ 7	10	+ 8
+ 5·5 to +14·5	− 1	+ 6	+ 5	12	− 7	− 1	20	+ 2

be indicated automatically by the *differences* between exposures. Its effect on the *mean* of the exposures is practically insensible, for its mean value is as follows:—

Exposures at X=.				Separate Values.				Mean.
−15	−6	+ 3	+12	+7	+1	0	+5	+3·3
−14	−5	+ 4	+13	+7	+1	+1	+6	+3·8
−12	−3	+ 6	+15	+5	+0	+1	+7	+3·3
−11	−2	+ 8	...	+4	0	+2	...	+2·0
−10	−1	+ 9	...	+3	0	+3	...	+2·0
− 9	0	+10	...	+3	0	+3	...	+2·0
			etc.			etc.		

We may remark that if there were a corresponding error in the X coordinates, which would not be detected in the same way, the amount of it would only range from 0 to +·0007 at the edge of the plate, and by excluding the two outside rows this could be reduced to +·0004. Taking the mean value over the plate as +·0002 we should only be liable to errors of ±·0002 or 0″·06. See also § 21.

40. But it would be more satisfactory to trace the error to its source. It cannot arise from—

(1) *Tilt of the plate.*—This is of the form $KXY + LY^2$ in the Y coordinate, and contains no term in X^2.

(2) *Error of reduction.*—In converting standard coordinates into X and Y we have the formula

$$Y = \eta - AX^2 + \text{etc.},$$

and if the coefficient A were wrongly adopted we should get an error of the form under consideration. But the values of A for consecutive zones are

		Diff.
Zone +30°	A = ·0003149	·0000065
Zone +29°	A = ·0003084	·0000070
Zone +28°	A = ·0003014	

To explain our coefficient ·0000033 in this way we should thus require an error in plate centre of half a degree.

(3) *Curvature of réseau lines.*—This is easily checked by turning one plate upside down on another, film to film, and measuring differences in y across the plate. Adding the differences in 0 and 25, 1 and 24, 2 and 23, etc, etc., we eliminate any accidental error of orientation, and double any curvature effect, which is already once doubled by the reversal. The results of a single set of measures on the middle line and the outside lines are here given:—

X lines.	0	1	2	3	4	5	6	7	8	9	10	11	13
	25	24	23	22	21	20	19	18	17	16	15	14	12
Y=13 & 13	0	−5	−11	+1	+10	−2	−1	+3	−1	+1	+3	+2	+1
Y= 0 & 26	+2	−1	+ 3	−2	−11	+4	−1	−3	+2	+3	0	−2	+10
Sum	+2	−6	− 8	−1	− 1	+2	−2	0	+1	+4	+3	0	+11

(The unit is 0″·03.)

The "Sum" contains eight times any curvature correction, and it is clear that the source of the error under investigation is not to be found in the *réseau* lines, which seem to be particularly straight.

(4) *Distortion.*—It is difficult to ascribe the errors to optical distortion. If this makes lines curve we should expect the curvature to have its convexity opposite ways on opposite sides of the centre, and thus to disappear in the mean of top and bottom. Moreover, a glance at the figures given in *Mon. Not.*, lix. p. 443, will show that the object-glass was at any rate free from such distortion at that date (though, of course, it may have altered since), the mean Y residual at $x = \pm 10$ being $0''{\cdot}00$ and at $x = \pm 4$ being $-0''{\cdot}01$, whereas the above formula gives a difference of $\cdot 0000033\ (100 - 16) = \cdot 00028 = 0''{\cdot}08$.

At present the source of error must be left frankly unidentified. Its effects on the mean places are minute, and its interest lies rather in showing the powerful scrutiny exercised automatically by the method than in any sensible improvement of the results.

41. The discussion of the Y residuals themselves (as distinguished from the differences $Y_n - Y_{n+1}$) has at present raised no point of special interest; and as this paper is already long, it may conveniently be restricted to this review of possible systematic errors, leaving the detailed discussion of residuals from several plates for a later communication. A brief summary of the situation may now be given.

Summary.

A. A method is suggested of photographing many consecutive regions of the same declination and hour angle on the same plate, with considerable overlap (§ 2 and § 7).

B. The reduction of such a plate is easy when once detailed tables have been formed, as for the Oxford zones (§ 8–11).

C. The mean differences between exposures give us ultimately the scale value (§ 9 and §§ 13, 14) and orientation of the plate (§ 10 and §§ 35, 36) from known stars near the beginning and end; and then the plate centres (§ 25 and § 37).

D. Subtracting these mean differences from the individual differences, we get material for discussion of systematic errors. The scrutiny is a powerful one, and is capable even of magnifying the effect on the mean results. Thus an error of the form AX^2 may appear in the scrutiny as $0''{\cdot}18$, whereas its greatest effect on the mean star place is $\pm 0''{\cdot}06$ (see § 21 and § 39).

E. No systematic error has been brought to light which need affect the results to an extent comparable with known sources of error in the Astrographic Catalogue. The most serious was almost certainly due to the use of a superimposed *réseau* (on a separate plate), and disappeared when this use was given up (§§ 16–22). An unexplained but small error exists in the Y

measures (§§ 39–40). On the other hand, search for errors of tilt (§§ 17, 18), *réseau* lines (§§ 19, 40), reduction (§ 14, § 40), and magnitude equation (§ 31), all gave satisfactorily negative results.

F. The main discussion of final places is reserved for a future paper, the present paper being restricted to the consideration of systematic errors. But an incidental result is interesting. It appears that the well-known magnitude equation of the Cambridge meridian observations changed character sensibly about 1879·0 (§§ 6, 29, 30.)

A Critical Comparison of the Overlapping Section of the Oxford and Potsdam Astrographic Catalogues. By G. D. C. Stokes, M.A., B.Sc., Lecturer in Mathematics in the University of Glasgow.

(*Communicated by Professor H. H. Turner.*)

1. *Introductory.*—It is possible to find the rectangular co-ordinates of a star on an Oxford plate ($A^h + 9^m$, 31°) from those of the same star on the adjacent plate (A^h, 31°) in at least two ways:—

(i). We may calculate the standard coordinates ξ_1, η_1 from the measures x_1, y_1 on (A^h, 31°) by using the constants given in the Oxford catalogue; then find ξ_2, η_2 for the second plate from ξ_1, η_1 by pure geometry; finally obtaining x_2, y_2 from ξ_2, η_2 by using the Oxford constants for ($A^h + 9^m$, 31°).

(ii). We may solve the overlapping Potsdam plate ($A^h + 5^m$, 32°) with both Oxford plates directly from the measures, thus obtaining x', y' (the Potsdam measures) in terms of x_1, y_1, and also of x_2, y_2, whence a simple elimination gives the result.

The first method involves errors of the Oxford provisional constants (which themselves involve errors of the photographic places, errors of the Cambridge or Leiden meridian places of the stars used for their determination, and errors of some 25 years' proper motion). The second method involves only errors of Potsdam and Oxford measures, along with errors of some 8 years' proper motion. The question arises: to what extent do the two solutions agree, and to which sources are the discrepancies mainly due? The present paper proposes to deal chiefly with this question.

2. *Solution for the Potsdam-Oxford constants.*—The following table shows the number of plates dealt with, their dates and numbers of stars:—

Plate No. Pots.	R.A. of Oxford Centre.	Date. Pots.	Interval. O—P.	Total No. of Stars. Pots.	Total No. of Stars. Oxf.
	h m		y		
39	5 37	1899·0	4·8	809	1071
	5 46		4·8		910
40	5 46	1899·0	4·8	760	910
	5 55		4·8		629
126	18 40	1893·5	7·2	533	944[1]
	18 49		7·2		1183[2]
129	19 7	1893·6	7·1	426	1235[3]
	19 16		8·9		1341
131	19 25	1893.6	9·0	704	1920
	19 34		7·1		941
132	19 34	1893·6	7·1	631	2180[4]
	19 43		8·9		2208
134	19 52	1893·6	8·9	732	2119
	20 1		9·1		1335
135	20 1	1893·6	9·1	573	1335
	20 10		8·0		1225
137	20 19	1893·6	8·0	439	1324
	20 28		8·0		1308
138	20 28	1893·6	8·0	381	1308
	20 37		9·1		685
139	20 37	1893·6	9·0	652	685
	20 46		9·0		1015
140	20 46	1893·6	9·0	570	1015
	20 55		9·1		719
141	20 55	1893·7	9·0	650	719
	21 4		9·0		663
142	21 4	1894·6	8·1	590	663
	21 13		8·1		453
143	21 13	1893·7	9·0	462	453
	21 22		9·0		720
144	21 22	1894·6	8·1	323	720
	21 31		8·1		654

1, 2, 3, 4. The numbers actually catalogued are 495, 535, 584, 942.

These plates cover the richest regions of the zone, and only a few equally good ones (*e.g.* No. 136) have been omitted—in some cases because the Potsdam plate was not then available.* The Oxford plates are later, and show a considerable excess in number of stars, so the Potsdam stars were identified in Oxford. For purposes of solution 40 stars distributed all over the overlapping area (but not too near the edges), and of magnitude generally between 8·5 and 10·0 (Potsdam scale), including, therefore, stars neither too bright nor too faint to give good images, were selected from Potsdam. The Potsdam coordinates of all the rest (hereafter

* I am indebted to Prof. Scheiner for the privilege of using several plate sheets before their publication in vol. v.

called the "non-solution" stars) estimated to appear in Oxford were written on a separate sheet and the whole then identified. The solution was conducted on lines similar to the Oxford solution for the provisional constants of the catalogue, except that the relations assumed were

$$x + \mathrm{P} = (1 + a')x' + b'y' + c'$$
$$y + \mathrm{Q} = d'x' + (1 + e')y' + f'$$

and the second order terms P, Q (0, 1 and occasionally 2 in the third decimal) were applied to every star from hyperbolic diagrams before solving the 40 equations. The values of the constants are given in the adjoined table. For convenience the series is divided into two sets—(i) the α series: $(A^h, 31°)$ solved with $(A^h + 5^m, 32°)$; constants a'_1, b'_1, etc.; (ii) the β series: $(A^h + 9^m, 31°)$ solved with $(A^h + 5^m, 32°)$; constants a'_2, b'_2, etc.

On the whole, the expectation that the solution from so many selected stars would be the best has not been justified. The "correction" of the constants is discussed in § 12; the results at present refer to the "uncorrected" solution.

Constants for the Oxford-Potsdam Solution.

	Plate No. (Potsdam).	*a*.	*e*.	*b*.	*d*.	*c*.	*f*.
		− ·000	− ·000	− ·00	+ ·00	+25	+24
α Series.	39	59	63	934	962	·4504	1·1408
	40	68	79	561	553	·8015	1·0113
	126	71	62	523	511	·1187	·8174
	129	76	63	224	238	·2208	·8644
	131	59	75	489	518	·4403	·6670
	132	48	56	421	432	·1470	·8679
	134	69	76	596	621	·5401	·6474
	135	67	69	864	860	·5037	·7664
	137	53	76	1128	1147	·6493	·6747
	138	76	66	363	329	·7945	·4610
	139	64	63	753	760	·4943	·7123
	140	49	72	553	567	·3155	·7801
	141	59	58	854	873	·8319	·5358
	142	49	62	652	652	24·0705	·9560
	143	52	72	786	795	25·2226	·5230
	144	65	58	574	569	25·1709	·6586

Constants for the Oxford-Potsdam Solution—continued.

	Plate No. (Potsdam).	*a.*	*e.*	*b.*	*d.*	*c.*	*f.*
		− ·000	− ·000	+ ·0	− ·0	+	+
β Series.	39	63	68	1155	1148	2·5866	25·1871
	40	66	69	1232	1208	3·0545	25·0140
	126	65	60	1421	1448	2·2523	24·6363
	129	89	85	2116	2105	2·3635	24·6881
	131	93	75	1472	1506	2·1317	25·1002
	132	61	93	1311	1282	2·7067	24·5982
	134	42	79	1254	1247	2·5465	24·7957
	135	73	98	1521	1521	2·7914	24·8003
	137	75	84	1715	1720	2·8139	24·6568
	138	78	59	1176	1184	2·7126	24·6511
	139	71	79	1500	1493	2·6170	24·8804
	140	62	67	1108	1115	2·4534	24·7756
	141	72	91	1732	1697	1·7590	24·8061
	142	74	64	0985	0964	2·1851	24·7131
	143	88	77	1663	1643	2·4980	24·5690
	144	63	55	1635	1627	1·2978	25·0871

3. *Residuals.*—

$$R(x) = x + P - [(1 + a')x' + b'y' + c']$$
$$R(y) = y + Q - [d'x' + (1 + e')y' + f'].$$

These were calculated to four decimals, of which the last was rejected in the result. Of the two examples appended, 126 represents a normal result, and 131 β is the most abnormal in the series.

Residuals and Probable Errors.

(*Unit* ·001 *Réseau Interval.*)

	Solution Stars.								Non-solution Stars.							
	126α.		126β.		131α.		131β.		126α.		126β.		131α.		131β.	
R.	*x.*	*y.*	*x.*	*y.*	*x.*	*y.*	*x.*	*y.*	*x.*	*y.*	*x.*	*y.*	*x.*	*y.*	*x.*	*y.*
−6	..		..		..		..		1		..		..		2	
−5	..		..		..		..		1		1		1		6	
−4	..		..			1		..	3	1	4	1	6	2	4	4
−3	1	3	3	0	1	2	2		5	0	8	5	9	2	13	6
−2	7	3	4	4	4	6	2	6	15	12	15	16	20	13	37	21
−1	3	7	7	13	10	7	8	12	13	18	22	26	43	18	37	25
0	13	16	7	10	10	8	12	8	19	20	23	24	33	25	29	38
+1	11	5	14	8	7	10	12	6	13	18	15	18	20	32	14	31
+2	4	5	4	4	6	4	2	6	7	12	14	15	8	28	11	21
+3	1	1	1	1	2	1	1	1	4	2	5	2	4	19	8	9
+4	..		..			1		..	2		..		1	7	3	7
+5	..		..		..		..		..		..		2	2	0	2
+6	..		..		..		..		..		..		1		1	1
Sum.	40	40	40	40	40	40	39*	39*	83	83	107	107	148	148	165	165
P.E.	±·9	±1·0	±1·0	±·8	±1·0	±1·1	±·8	±·9	±1·3	±·9	±1·2	±1·0	±1·2	±1·3	±1·5	±1·3

* One star rejected for proper motion.

It is apparent that while the solution stars for 131β show a very good consistency, a prominent magnitude equation is indicated by the non-solution stars. As the same feature appears in the adjacent Oxford plate (131α) its origin is probably in the common Potsdam plate. The adjoined table gives the means (in units of ·001 *réseau* interval, or o″·3) for both solution and non-solution stars, and in brackets the numbers of stars on which the respective means depend.

Mean Residuals.

(*Unit* ·001.)

	α Series.			β Series.		
R(x)	− ·62 (45)	− ·56 (88)	− ·12 (32)	− ·13 (47)	− ·16 (69)	− ·29 (59)
	− ·28 (208)	− ·65 (255)	− 1·00 (184)	− ·83 (211)	+ ·15 (234)	− ·28 (160)
	+ ·33 (66)	+ ·23 (133)	+ ·25 (69)	− ·10 (70)	+ ·09 (112)	+ ·08 (76)
	+ ·80 (198)	+ ·31 (249)	+ ·04 (212)	− ·14 (324)	− ·63 (237)	− ·12 (176)
	− ·16 (56)	+ ·03 (95)	+ ·17 (54)	− ·18 (60)	− ·14 (93)	+ ·14 (49)
	+ ·78 (95)	+ ·07 (158)	+ ·03 (174)	+ ·15 (261)	− ·30 (183)	− ·57 (103)
R(y)	+ ·24 (45)	+ ·26 (88)	− ·22 (32)	− ·06 (47)	+ ·36 (69)	− ·08 (59)
	+ ·35 (208)	+ ·61 (255)	+ ·02 (184)	+ ·66 (211)	+ ·47 (234)	+ ·42 (160)
	− ·11 (66)	− ·17 (133)	− ·04 (69)	+ ·10 (70)	− ·09 (112)	0 (76)
	+ ·41 (198)	− ·17 (249)	− ·70 (212)	− ·03 (324)	− ·33 (237)	+ ·08 (176)
	− ·12 (56)	0 (95)	− ·06 (54)	+ ·30 (60)	+ ·15 (93)	− ·08 (49)
	+ ·47 (95)	+ ·29 (158)	− ·01 (174)	− ·19 (261)	− ·02 (183)	+ ·42 (103)

Taking the plate as a whole, the means over the whole series are—

			+	−	No. of Stars.	Mean.
R(x).	α series.	Solution.	3·55	3·63	638	− ·01
		Non-solution.	11·83	12·58	1733	− ·04
R(x).	β series.	Solution.	3·64	3·77	635	− ·02
		Non-solution.	11·45	14·95	1889	− ·18
R(y).	α series.	Solution.	3·35	3·50	638	− ·02
		Non-solution.	12·66	10·56	1733	+ ·12
R(y).	β series.	Solution.	3·36	3·30	635	+ ·01
		Non-solution.	13·46	11·13	1889	+ ·12

There is, therefore, some evidence for the conclusion that

relative to the brighter stars (about 25 per cent. of the whole number) the stars on the Oxford plates, as a whole, are displaced in the directions of decreasing R.A. and increasing Dec. From the distribution it appears that the upper region of the plate is most abnormal.

The residuals as a whole show that the probable error of a coordinate calculated from the constants of the forty stars solution is $\pm 1{\cdot}0$ ($0''{\cdot}3$) for a solution star, and $\pm 1{\cdot}3$ ($0''{\cdot}4$) for a non-solution star; and that from three to four per cent. of the residuals exceed 3·0 numerically for solution stars and exceed 4·0 numerically for non-solution stars. Residuals exceeding these latter limits may be attributed to bad solutions, inferior measures or star places, proper motions, and errors.

It is interesting to compare the probable errors for the queried cases in the Potsdam catalogue. Of 180 stars marked as having doubtful measures, four show an x residual greater than 6, namely—

Plate.	No.	Mag.	R(x).	R(y).	Potsdam mark.
134	16	10·0	+12	+6	mess. sehr unsicher.
40	759	10·5	−11	+6	??
40	621	10·2	+8	−2	??
143	370	10·0	−7	−1	?

If we reject these four only (and it must be remembered that the discordances of § 6 were rejected in calculating the P.E. for non-queried cases), the P.E. of the rest comes out $\pm 1{\cdot}3$ for both x and y, *i.e.* precisely the mean value for all the solution stars. Any anxieties on the part of the Potsdam measurers as to the inferior value of the queried measures are therefore groundless.

In addition there are 82 doubtfuls (9 identified as spots on the plate) for which there are no stars in Oxford. Of these 5 had magnitude between 9·0 and 9·5 (all found to be spots), 15 between 9·5 and 10·0, and 62 fainter.

4. *Errors.*—Out of some 5000 stars (including one or two small plates not used here) 19 presumed cases of error were found, of which 15 were acknowledged as due to Potsdam, 3 to Oxford (previously discovered), and one turned out to be a proper motion (P.M. list 140β, Nos. 332, 333). The corrections need not be given here, as they are known at both observatories concerned.

5. *Overlaps for the same catalogue.*—Stars on the overlapping strip of two Oxford plates afford a test of the solutions with the common Potsdam plate, and also furnish evidence with regard to measures and distortion. Let the residual from the α pair of plates be R(α); then for perfect conditions R(α) should equal R(β). Excluding discordances, the sums of R(α) − R(β) for the overlaps are as follows (A, B, C denoting the upper, middle, and lower divisions of the strip).

	x No. of Stars. +	−		*y* No. of Stars. +	−	Weighted means. *x*	*y*
			Oxford Overlap.				
A	113	44	136	34	132	+0·5	−0·7
B	77	81	194	45	105	0	−0·4
C	77	40	123	48	40	+0·2	+0·1
Sums	267	165	453	127	277		
			Potsdam Overlap.				
A	33	25	62	28	15	+0·2	+0·2
B	42	22	66	46	17	+0·4	+0·6
C	28	4	17	8	12	+0·6	−0·1
Sums	103	51	145	82	44		

Comparison with the distribution of residuals on p. 114 shows at once that the systematic differences here are largely due to inferior solution. The plus x value in A is the only one in disagreement with this assumption, and may reasonably be attributed to uncertain measures and distortion, in view of its position in the far corner. The distortion hypothesis requires (i) in x $R(\alpha) - R(\beta)$ of constant sign, (ii) in y $R(\alpha) = R(\beta)$, and each increasing numerically with y. It is clear, then, that errors of distortion are lost in the greater errors (about 0″·2), due to plate constants deduced from the brighter stars. Except in the extreme corner of the plate, therefore, the overlaps show no evidence of distortion exceeding 0″·1.

6. *Discordances.*—Some critical information on the question of proper motion is furnished by the following tables, comparing the residuals of the same star on adjacent plates. The results are amply confirmed by the independent calculation of the R.A. and Dec. directly from the catalogues.

Two Oxford Images with one Potsdam Image.

Unit ·001.

Plate No. Pots.	No. of Star. Pots.	Oxf.	Mag. Pots.	Interval. *y*	R.A. h m s	Dec. ° ′ ″	Δ*x*.	Δ*y*.	R(*x*).	R(*y*).	Discordance. *x*.	*y*.
134α		54569		8·9		28·35		23·8	− 1	− 1	− 1	− 2	+ 1	− 8
	305		10·8		19 56 28·38	54 24·2								
134β		55989		9·1		28·32		25·8	− 3	+ 5	− 2	+ 6	+ 2	− 6
137α		58425		8·0		55·18		37·0	+ 3	+ 1	+ 1	− 1	+ 6	− 4
	175		10·0		20 22 55·12	26 36·6								
137β		59726		8·0		55·04		38·0	− 4	+ 5	− 5	+ 3	+ 7	− 4
137α		58476*		8·0		44·12		49·5	+ 3	+ 1	0	− 1	0	− 6
	232		9·0		20 23 44·06	31 49·1								
137β		59786*		8·0		44·09		50·9	+ 1	+ 6	0	+ 5	+ 2	− 5
143α		64018		9·0		39·67		58·8	+ 2	− 1	+ 2	− 2	+ 8	− 4
	226		9·5		21 17 39·64	53 59·1								
143β		64735		9·0		39·49		60·5	− 7	+ 5	− 6	+ 2	+ 9	− 6

(The R.A. and Dec. in the middle line refer to the Potsdam star.)

An apparent x discordance of ·013 in Pots. 172 with Oxf. 64540 and 65301 was found to be due to an error of ·010 in the former Oxf. star. In 59726 above, the image is bad and elongated; in 59786* there was found a small error in y of ·004. The others were unchanged on revision. There is thus little more than a suggestion of systematic error here, +·004 in x and −·006 in y at the outside.

Two Potsdam Images with one Oxford Image.

39β	(751)		9·2	4·8		6·21		7·8	+ 1	− 1	+ 1	0	+11	+ 6
		13964			5 46	6·20	8	7·6						
40α	33		9·7	4·8		5·95		6·0	+13	+ 5?	+12	+ 6	+12	+ 6?
131β	(704)		9·3	7·1		58·45		20·4	−12	+ 7	− 6	+ 1	+ 8	− 1
		50037*			19 34	58·20	36	22·4						
132α	35		9·0	7·1		58·17		22·1	+ 2	+ 1	+ 2	0	+14	− 6
131β	653		9·0	7·1		54·51		37·6	+ 1	+ 6	+ 3	+ 1	+ 7	+ 2
		49992*			19 33	54·52	31	39·4						
132α	2		9·5	7·1		54·30		38·4	+11	+ 3	+10	+ 3	+10	− 3
134β	692		9·7	9·1		53·34		31·8	− 5	+ 1	− 3	+ 1	+11	− 1
		56060			20 0	53·23	59	32·2						
135α	2		10·5	9·1		53·04		31·8	+ 9	+ 1	+ 8	0	+14	0
137β	(402)		10·0	8·0		55·85		28·7	− 1	− 1	− 1	− 1	+ 8	+ 1
		59867			20 27	5584	33	28·5						
138α	2		10·5	8·0		55·56		28·3	+14	+ 1	+ 7	0	+15	+ 2
137β	417		11·0	8·0		28·38		11·1	− 4	+ 2	− 4	+ 2	+ 8	0
		59750			20 28	28·29	26	11·7						
138α	(16)		10·0	8·0		28·10		11·0	+ 9	+ 2	+ 4	+ 2	+13	0
137β	435		10·5	8·0		59·27		7·0	− 8	+ 2	− 8	+ 2	+ 8	− 3
		59755			20 28	59·11	26	7·6						
138α	(35)		9·2	8·0		59·00		7·7	+ 5	− 1	0	− 1	+13	− 3
138β	371		11·0	9·1		42·16		26·2	−16	+12	−10	+14	+ 8	−17
		60616			20 37	41·84	57	29·9						
139α	23		9·0	9·0		41·86		31·0	− 1	− 4	− 2	− 3	+15	−16
141β	649		11·0	9·0		2·79		5·0	− 8	− 1	− 5	− 2	+ 6	+ 2
		63442			21 5	2·62	43	4·6						
142α	49		11·0	8·1		2·63		3·3	0	+ 4	+ 1	0	+ 8	+ 5

(The R.A. and Dec. in the middle line refer to the Oxford star, and bracketed Potsdam numbers indicate solution stars.)

Δx, Δy are the catalogue differences (Oxf.—Pots.) in units of ·001 *réseau* interval. R(x), R(y) are the values from the solution in the present paper and should agree with Δx, Δy. The discordance column gives R(α) − R(β), the lower number referring to the direct catalogue confirmation. The large y discordance in 371 (Pots.) suggests a case of error and is perhaps better rejected. It seems probable, however, that the systematic x discordance (mean = +·011) is genuine. We have seen that this cannot be attributed to distortion. Again, these stars constitute only a

fraction of the overlapping stars, and must be regarded as exceptional cases among them.

7. *Proper motions.*—The following list includes all cases worthy of serious consideration under this head:—

Plate No. (Pots.).	R.A. of Oxford Centre.	No. of Star. Pots.	No. of Star. Oxf.	Mag. (Pots.).	Δx.	Δy.	Interval.	Centennial P.M. R.A.	Centennial P.M. Dec.	Evidence from Oxf.-Leiden Comparison, and Remarks.
							y	″		
126α	18 40	202	43996*	6·0?	+ 6	+ 1	7·2	+25	...	No p.m.
126β	18 49	202	44493*	6·0?	+ 6	− 1	7·2	+25	...	
126α	18 40	232	43837	10·5	− 2	+ 7	7·2	...	+ 29	
126β	18 49	232	44315	10·5	− 2	+ 7	7·2	...	+ 29	
129α	19 7	16	45653	10·3	−12	+ 1	7·1	−51		
129α	19 7	226	45728	9·3	− 9	0	7·1	−38		
129β	19 16	226	46879	9·3	−11	0	8·9	- 37		
129β	19 16	259	47082	10·5	+10	+ 2	8·9	+34		
129β	19 16	424	46658*	10·0	−12	+ 8	8·9	−40	+ 27	No p.m.
131α	19 25	19	48567*	8·7	+ 2	−11	9·0	...	− 37	−48″ (dec.), but Leiden differs from Bessel by 20″.
131β	19 34	378	49867	10·2	+12	+ 1	7·1	+51		
132α	19 34	(89)	49778	10·5	+ 2	+ 8	7·1	...	+ 34	
132α	19 34	(285)	49750	10·2	+ 1	+ 6	7·1	...	+ 25	
132β	19 43	285	51364	10·2	+ 2	+ 4	8·9			
132α	19 34	23	49744	10·5	+ 7	+11	7·1	+30	+ 46	
132β	19 43	(572)	52271*	10·0	+12	−10	8·9	+40	− 34	+47″, −41″.
132β	19 43	544	52473	9·2	−13	− 2	8·9	−44		
134α	19 52	16	53645	10·0	+12	+ 6	8·9	+40	+ 20	Pots. "sehr unsicher" but Oxf. image good.
135α	20 1	111	55414*	7·5	− 1	+ 6	9·1			See note (i).
		112	55415	9·5	− 5	− 1				
137β	20 28	(368)	59960	10·0	0	− 6	8·0	...	− 22	
137β	20 28	(373)	59861*	9·8	+ 4	+ 5	8·0	+15	+ 19	+17″ (Dec.).
139β	20 46	545	61935*	8·5	− 4	− 9	9·0	...	− 30	−30″ (Dec.).
140β	20 55	332	...	11·0	− 2	−38	9·1	...	−123	See note (ii).
		333	62702	9·0	− 2	−36				
140β	20 55	436	62799	8·5	+15	0	9·1	+50		
141α	20 55	27	62468	11·0	+ 8	+ 7	9·0	+27	+ 33	
141α	20 55	95	62636	11·0	+10	− 2	9·0	+33		
141α	20 55	220	62728	10·0	+ 8	0	9·0	+27		
141α	20 55	327	62819	9·0	+ 7	0	9·0	+23	...	Near corner of Oxf. plate.

Plate No. (Pots.).	R.A. of Oxford Centre.	No. of Star. Pots.	No. of Star. Oxf.	Mag. (Pots.).	Δx.	Δy.	Interval.	Centennial P.M. R.A.	Centennial P.M. Dec.	Evidence from Oxf.-Leiden Comparison, and Remarks.
							y	"	"	
141β	21 4	(622)	63213	9·5	− 1	+ 9	9·0	...	+ 30	See note (iii).
142β	21 13	462	63806	9·2	+ 3	−34	8·1	...	−125	
143β	21 22	(430)	64449*	9·5	− 9	− 2	9·0	−30	...	No p.m.
143β	21 22	343	64494	10·2	0	− 7	9·0	...	− 23	
143β	21 22	370	64640	10·0	− 7	− 1	9·0	−23	...	Pots. "?".

Note (i).—A plate was taken in June 1910 specially to test the apparent relative motion in this double (111, 112 on 135α). The coordinates of the comparison star relative to the bright one are for Potsdam (1893·6) ·084, ·050; for Oxford (1902·7) ·080, ·043; for Oxford (1910·5) ·080, ·044. The evidence of the catalogue positions is therefore doubtful.

Note (ii) (332, 333 on 140β).—The faint star here is not given in the Oxford catalogue. The approx. measures are $x = 3'190$, $y = 22'775$, which give good evidence of large p.m. of the whole double, and not of error, as was at first supposed.

Note (iii).—This star (622 on 141β) should also occur on the Potsdam plate 142 at −11·35, −12·61, but there is no star catalogued there.

How many of these cases would survive critical examination it is hard to say. Certainly the preceding list of discordances should serve to engender an attitude of caution in fixing any lower limit for proper motion on the above evidence alone. Circumstances prevent further investigation of these interesting cases, but it is hoped that the list may be found useful to others engaging in a more extended survey of the problem.

We may note, however, that in Professor Turner's "Second Note on the number of faint stars with large proper motion" (*Mon. Not.*, lxix. p. 495), 2 or 3 per 1000 stars out of 10,000 in zone +26° showed centennial P.M. exceeding 20″, and 6 or 7 per 1000 exceeding 15″, for an interval of 14 years. The proportion in the present paper for 4900 stars in zone +31° is 6 per 1000 for the 20″ limit, the interval being only 8 years. The values of the P.M. here quoted are thus almost certainly too large, but investigation would very probably reduce both the number and value to correspond with the 26° results, and thus render their genuineness more probable.

Comparison of Magnitudes.

8. *Position Equation.*—From a family of curves representing $a - 1{\cdot}36\sqrt{d}$ the Oxford magnitude (m) was read from the catalogue d and the difference $m—m'$ assigned to the appropriate section of the plate for every star. The analysis of mean values gave the following results. There were only 7 stars with $m' < 7$, giving a mean residual −0·6; for m' between 7 and 8, 39 stars gave zero for average residual. The details for the fainter stars are given below. For the α series A is the Oxford centre, C that of Potsdam; for the β series, D is the Oxford and B the Potsdam centre. The number of stars on which each mean is based is also appended.

Mean Values of m—m′.

(Unit 0·1 mag.)

$m'=8$–9.

B	6+		C
	+2 (31)	+1 (42)	−4 (14)
α	0 (25)	+4 (40)	+1 (23)
	−2 (16)	−4 (27)	0 (17)
A		$n=235$	D

$m'=9$–10.

	14+		
	−2 (94)	+3 (144)	0 (96)
α	−3 (105)	+3 (154)	+4 (122)
	−6 (63)	−1 (98)	+3 (92)
		$n=968$	

$m'=10$–11.

	13+		
	0 (124)	+2 (159)	−1 (105)
α	−1 (131)	+3 (186)	+1 (138)
	−5 (74)	−1 (127)	+1 (118)
		$n=1162$	

B	4+		C
	−9 (27)	+5 (33)	0 (29)
β	−5 (47)	+6 (41)	+2 (33)
	−1 (33)	+3 (36)	−1 (13)
A		$n=292$	D

	13+		
	−6 (141)	+5 (117)	+2 (91)
	−2 (176)	+4 (154)	−1 (86)
	−2 (130)	+3 (114)	−3 (54)
		$n=1063$	

	13+		
	+1 (88)	+2 (136)	0 (99)
	−2 (172)	+3 (156)	+1 (134)
	−2 (159)	+2 (124)	−5 (82)
		$n=1150$	

Total number of stars (8–9) 527 ; (9–10) 2031 ; (10–11) 2312.

It is evident that the maximum *plus* is at the centre of the quarter plate in all cases, but the maximum *minus* is at the A or C corners in the α set and at the B or D corners in the β set. Further, if the α plate be divided up by parallels to B D, and the β plate by parallels to A C, there clearly appears a general drift through −0+0− from the Oxford to the Potsdam plate centre. The consistency in the α series is remarkable, but the β series shows a distinct minus value near A. Remembering that these numbers give the corrections to the mean value of (Oxf. mag.)—(Pots. mag.), and that the greater value of m corresponds to the smaller star image (d), we infer that near either plate centre the Oxford d is too large or the Potsdam d too small, and *vice versa* midway between plate centres. As regards Oxford magnitudes, these facts tend to confirm the result obtained by Professor Turner (*Mon. Not.*, lxii. p. 447) for the position of best focus of the Oxford equatorial (about 35′ from the centre).

9. *Magnitude Equation.*—The mean values of m—m' are representable by a linear function of m' up to $m'=9{\cdot}5$. Beyond that limit there is a pronounced defect amounting to more than

half a magnitude. The magnitude number-distribution is as follows:—

m'.	No. of Stars.
5– 7	7
7– 8	39
8– 9	527
9–10	2031
10–11	2312
Total	4916

The same falling off is thus apparent in the number of stars per Potsdam magnitude, and the numbers per Oxford magnitude show the same tendency even more strongly. This arises partly from the fact that in the richer regions here considered a large number of Oxford stars, though measurable, were not measured, partly because the smallest estimated diameter (about 8 or 10) corresponds to $m = 12{\cdot}0$ on some plates, and to $m = 13{\cdot}3$ on others, and again a considerable number of images would be at the limit of visibility.

The simplest comparison formula is therefore

$$m - m' = 0{\cdot}63\ (m' - 7{\cdot}5) + 0{\cdot}4f$$

where f varies from -1 to $+1$ according as the star is near a plate centre or near the region midway between plate centres. For $m' > 9{\cdot}5$ the value $9{\cdot}5$ must be used.

10. *Anomalies.*—There are two stars for which the Potsdam and Oxford magnitudes differ by 3 magnitudes.

Plate.	No. of Stars.			Magnitude.				Reviser's remark.
(Pots.)	Pots.	Oxf.	Leiden.	Pots.	Oxf.	Leid.	B.D.	
132β	246	51596	7579	8·7	5·4	8·8	7·8	Oxf. diam. too large.
129a	23	45614	7170	8·0	5·0	6·0	6·0	Oxf. diam. might even be larger.

In the first case the mean difference $m - m'$ for 6 near stars of the same magnitude is $-0{\cdot}1$ and the formula gives $m = 9{\cdot}2$ from $m' = 8{\cdot}7$. In the second case there are no other near stars of the same brightness, but the table of mean values for 8–9 magnitudes shows that the expected $m - m'$ is $+0{\cdot}4$ instead of $-3{\cdot}0$.

The number of stars for which $m - m'$ exceeds 2 magnitudes is relatively very great, but the large majority of these are fainter than the 9th magnitude, and the catalogues differ systematically by nearly a magnitude and a half for the faint stars.

There are 22 Potsdam stars with m' from 9·0 to 10·0 which do not appear in the Oxford catalogue, though in every case a considerable number of fainter stars are given. Only 3 of the same class were found to be spots.

A few other typical cases of non-appearance are appended.

(i) 135*a*, 7, $m' = 9{\cdot}0$, = Oxf. 55910 ($d = 32$) = Leid. 7927 (mag. $9{\cdot}1$) = B.D. 31° 3929 (mag. $9{\cdot}1$). This star should certainly appear on the preceding Potsdam plate 134β at $-12{\cdot}712$, $-3{\cdot}560$.

(ii) 137*a*, 81, $m' = 9{\cdot}0$, = Oxf. 18·443, 16·760 ($d = 18$). The very close Oxf. neighbour 18·432, 16·764 ($d = 15$), which looks very like its double, is not given in Potsdam.

(iii) 141β, 642, $m' = 11{\cdot}0$ } The equal mags. of this double (?)
643, $m' = 11{\cdot}0$ } justify the expectation of two Oxf. stars, but only one appears (63405) and it agrees with 642. No trace of the other star has been found on revision. Possibly the object is a small planet on Potsdam?

11. *Comparison of the two solutions.*— The Potsdam coordinates on plate ($A^h + 5^m$, 32°) have been determined in terms of Oxford coordinates x_1, y_1 and x_2, y_2 of both plates (A^h, 31°) and ($A^h + 9^m$, 31°) by direct comparison of the measures of 40 bright stars. By elimination x_2, y_2 are found as approximate linear functions of x_1, y_1, say

$$x_2 = A_1 x_1 + B_1 y_1 + C_1 + P_1$$
$$y_2 = D_1 x_1 + E_1 y_1 + F_1 + Q_1$$

where P_1, Q_1 are small second order terms. But the Oxford catalogue gives linear equations connecting x_1, y_1 and x_2, y_2 with the standard coordinates of the respective plates, and a relation (involving small second order terms) can be found between the standard coordinates on the two plates. Hence x_2, y_2 can again be found in terms of x_1, y_1, say

$$x_2 = A_2 x_1 + B_2 y_1 + C_2 + P_2$$
$$y_2 = D_2 x_1 + E_2 y_2 + F_2 + Q_2$$

in which only errors of the Oxford catalogue enter. The differences $\delta x, \delta y$ of these results are the object of investigation. The constants were calculated by the direct eliminations stated, and their values were confirmed by independent transformations, but the reductions need not be given here. In δx and δy the terms of second degree practically vanish. Finally, $\delta x, \delta y$ were corrected for magnitude equation in A_1, B_1, C_1 arising from the use of 40 bright stars in the original solution.

12. *Corrections of the Potsdam-Oxford constants.* — This correction makes the constants depend on the whole number of stars on the quarter plate (150 on an average). The quarter plate was divided into nine equal sections. The residuals of all the stars (excepting anomalies) were entered in their appropriate places and means were taken. In the following examples the corner number gives the number of stars in the division.

Mean Residuals.

(Unit ·0001 *Réseau* Interval.)

126α (y).

−7 (13)	+3 (14)	+4 (10)
−9 (13)	−1 (17)	0 (16)
0 (4)	+5 (17)	0 (5)

Sums +12 −17

132β (x).

−18 (32)	−10 (29)	−24 (18)
−1 (39)	−7 (34)	−13 (14)
+10 (32)	+8 (16)	−2 (7)

+18 −75

135β (x).

−26 (17)	−2 (25)	+1 (9)
−11 (25)	+11 (15)	−12 (6)
+1 (18)	+2 (18)	+8 (7)

Sums +23 −51

143β (x).

−18 (12)	−4 (16)	+5 (10)
−2 (19)	+8 (22)	+8 (14)
+10 (23)	+1 (12)	−7 (4)

+32 −31

The problem is to find a linear function of x, y which will leave smaller + and − sums of mean residuals and a better distribution when applied as a correction to these values.

Geometrically, the problem consists in drawing a plane to cut the plate, so that (i) R = 0 along the intersection, (ii) + and − values shall be on opposite sides, (iii) the numerically greatest values shall be furthest from the zero line. Practically only a compromise between these conditions can be attained. A comparison of the sums for all the plates showed that when Σ(R) did not exceed the +20 −20 range, the original solution was susceptible of little or no improvement. Six such cases were found; 20 showed only a small change, but all the remaining 38 were substantially improved.

126α (y) above is an instance of a good solution.

132β (x) is an abnormal case (note the P.M. in the solution star 572 (Pots.) +12, −10 in the third figure). Taking the origin at the left bottom corner it is easily seen that a plane making intercepts +12, +6, +12 on the axes, *i.e.* $z = -x - 2y - 12$, will correct the first solution. This reduces Σ(R) from +18 −75 to +13 −15, which is as good as can be got.

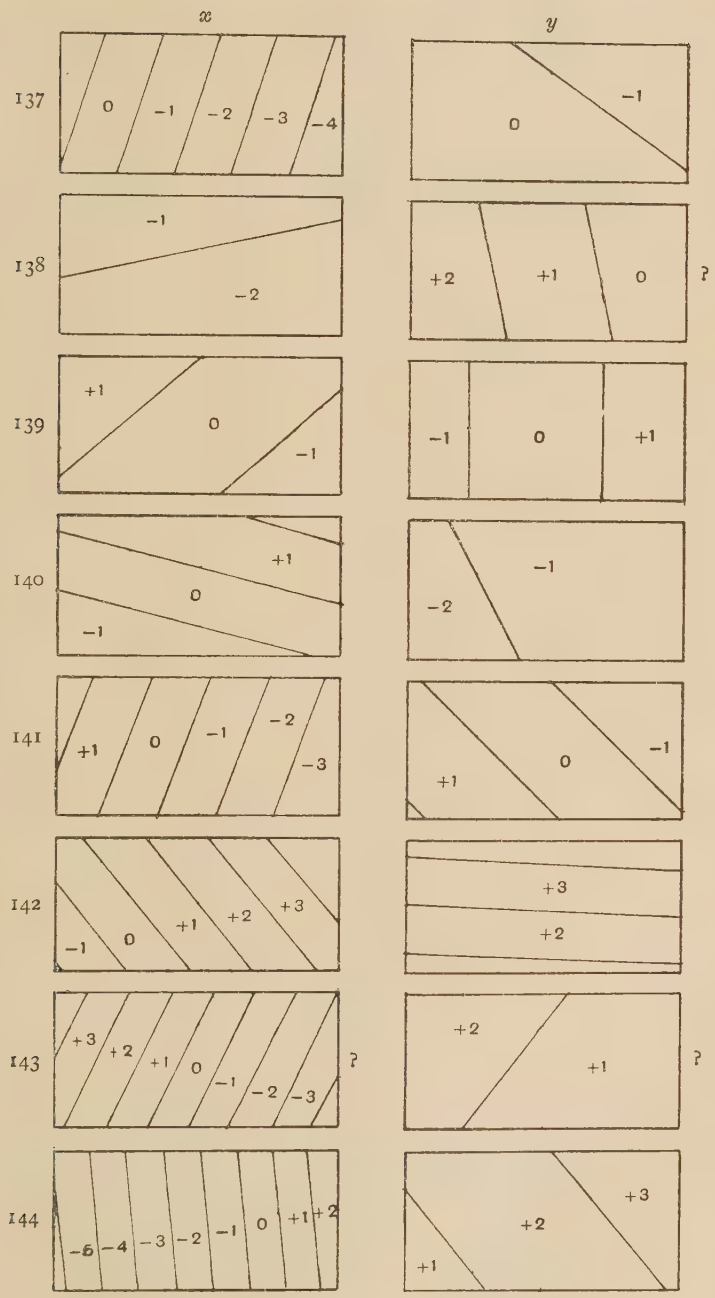
135 $\beta(x)$ is a difficult case. The correction $z = -4$ changes $\Sigma(R)$ from $+23-51$ to $+40-40$, while $z = 2x - 1{\cdot}6y - 20$ gives $+37-32$. Both are poor corrections, and the latter gives the apparent Oxford error a range of $-2{\cdot}5$ to $+5$ over the half plate (unit ·001), while the former give a range $-{\cdot}5$ to $+4$.

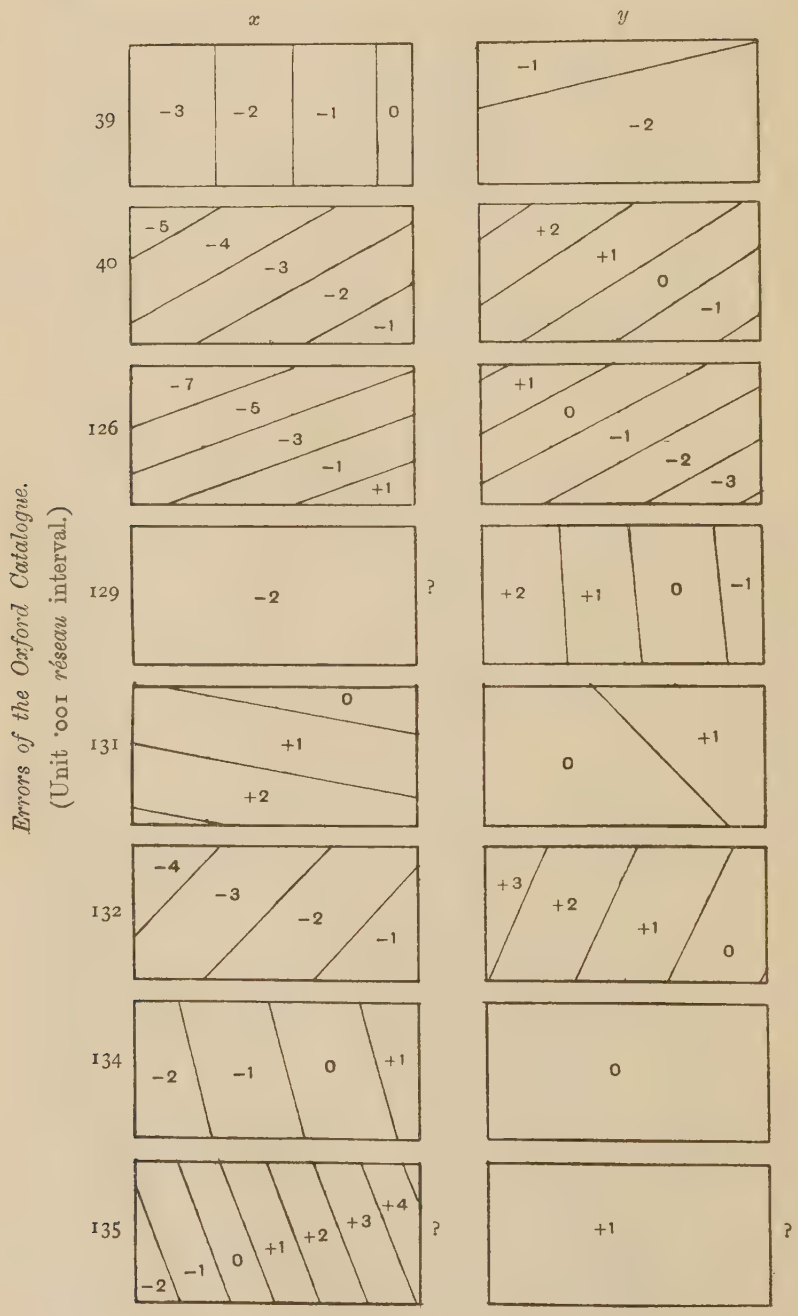
143 $\beta(x)$ exhibits some uncertainty. The -7 in the bottom right-hand corner depends on only 4 stars. No correction has been applied to this plate.

The mean ranges of the corrections over the quarter plate for the series are -8 to $+8$ in x and -5 to $+6$ in y. Or taking differences and doubling we have 32 in x and 22 in y for the half plate. Hence the solution of corner plates from 40 brighter stars is subject to an average accidental error ranging $1''{\cdot}0$ in x and $0''{\cdot}7$ in y *over a half plate.* The maximum range found is $2''{\cdot}7$ in x and $1''{\cdot}7$ in y.

13. *Errors of the Oxford Catalogue.*—The final values of δx, δy are exhibited graphically in units of the third decimal. The second order terms are too small to justify inclusion, especially in view of the doubt attached to a few of the corrections applied (*cf.* § 12, 135 $\beta(x)$). These diagrams cover the southern half of the Potsdam plate ($A^h + 5^m$, 32°) overlapped by the Oxford plates (A^h, 31°) and ($A^h + 9^m$, 31°). Compared with similar diagrams for δx, δy before correction for Potsdam-Oxford solution, they show distinct reduction in the magnitude of the apparent Oxford error for a few plates where the range of error was greatest (*e.g.* 131x, 134x, 144x, 39y, 132y), but in several others (135x, 126y) there is a slight increase. The average range before correction was $1''{\cdot}4$ in x, $0''{\cdot}8$ in y; after correction $1''{\cdot}2$ in x, and $0''{\cdot}5$ in y. The maximum range of error found is $2''{\cdot}4$ (126x). The existence of errors of the order of $1''$ attributable to the constants of the Oxford catalogue is thus clearly established. On the other hand, *these errors are no greater than the errors of corner plate solution from forty selected bright stars.* It must be remembered, however, that in scale value and in orientation the constants of the direct quarter plate solution can be only half as accurate as those of the catalogue whole plates, assuming equal number, distribution, and position error in the reference stars. On the average, the position errors to which these inferior constants give rise over the quarter plate will be the same as those of the catalogue constants extended over a whole plate. But in estimating δx, δy over the whole lower half of the Potsdam plate we are magnifying the errors of one or other of the quarter plate solutions. A similar error magnification *but smaller* (by reason of the larger catalogue plate) is made in deriving the relation between the sets of Oxford coordinates from the catalogue solution. Again, for the series of plates considered, the average number of meridian stars used in determining the catalogue constants is 60, and the number of observations of each of these stars is two or three compared with one photographic position. Therefore the photographic method with its magnitude equation of the same order as the catalogue

Errors of the Oxford Catalogue.
(Unit '001 *réseau* interval.)





Errors of the Oxford Catalogue.
(Unit ·001 *réseau* interval.)

errors arising from errors of meridian stars is subjected to a comparison with meridian observation unfair in respect of area of comparison, number of reference stars, and number of position determinations per star.

The practical result, however, is that the Astrographic Catalogue errors must be ascribed as much to erroneous photographic places as to errors of the meridian catalogue. The corrections of the 40 star solution, with reference to all the stars, are too great to sustain any other conclusion. In both catalogues the errors in R.A. are less than those in Dec.

My thanks are due to Mr. F. A. Bellamy for kindly examining the Oxford measures of all the stars in the discordance and P.M. sections, and also for the remarks in the latter section.

On the Errors of Measurements on Photographic Plates.

By Winifred Gibson.

(*Communicated by L. N. G. Filon.*)

1. *Introductory.*

Some years ago M. Loewy published three papers* on the accuracy of photographic measures, which gave rise to considerable discussion. Briefly, the more important conclusions he reached were as follows:—

1. In measuring the co-ordinates of star images on a photographic plate by means of *réseau* lines, a considerable systematic error occurs, which varies with the orientation of the plate, but is constant in any one orientation.

2. The total error, accidental and systematic, is best eliminated by measuring the plate in four orientations.

In the "systematic error" M. Loewy included the error of personality, which varies with the time and circumstances of the observation, and is not therefore truly systematic in the ordinary sense.

In the Supplementary Number of the *Monthly Notices* of the R.A.S. (1901), Mr Plummer rediscussed M. Loewy's figures, and concluded that a considerably better compensation of error is obtained by taking measurements in two opposite orientations of the plate, and therefore that the labour involved in taking four orientations is unnecessary.

The measurements on which the whole discussion was based were the Y co-ordinates of 82 different stars on a single plate. It seemed to me worth while to take many measurements of the same star in different orientations, and try to compare the magnitudes of the constant and accidental errors. With such measurements it would also be possible to find the amount of correlation,

* *Circulars of the Conférence Astrographique Internationale*, Nos. 8 and 9.

if any, existing between the accidental errors in different orientations of the plate. It is only by taking a large number of observations on a single image that it is possible to obtain a mean result practically free from accidental error. If any single star image be measured in four orientations once only, the mean of these four readings may quite probably be affected by a large accidental error, and any attempts to determine the best compensation for constant error based on the differences of the four readings from the mean of the whole would be futile.

Also, in order to find the correlation existing between accidental errors in different orientations, it is necessary either to know the true readings for a number of different stars, and correlate the differences of the four orientation readings from the true reading in each case; or, on the other hand, to make a large number of readings on the *same* star, subtract each reading from the mean of the whole, and correlate the results. It does not matter in this case if the mean of the whole is not the true mean, since the result of subtracting each reading from the mean of the whole will differ from the result of subtracting it from the true mean by a constant quantity, viz. the error of the mean. This constant quantity will not affect the correlation coefficient.

For both these reasons I decided to take readings of a single star image.

2. *Explanation of the method of work.*

The results given in this paper are based upon three different sets of observations. In each set a single star was chosen, and its X co-ordinate measured in four orientations of the plate. The measurements were obtained by making a setting first upon the left-hand *réseau* line on the plate, next upon the star image, then upon the right-hand *réseau* line, and finally again on the star image. The mean reading for the star image was subtracted from the reading for the left-hand *réseau* line, and the correction for run was applied. Each measurement was repeated in four orientations, the same distance being measured each time. Thus, taking opposite orientations, the distance was measured to the left in the first case, to the right in the second.

The first series of measurements consists of 110 observations of a single star made by myself with the plate-measuring instrument of the Cambridge Observatory.

The second series gives 80 observations made on another star with a different instrument. A different star image was chosen in this case, but the measurements were again made by myself. In order to test the effect of the particular instrument used on the errors of measurement a plate-measurer was made, similar in some respects to that of the Cambridge Observatory, but with considerable modifications.

The instrument consists of a microscope mounted upon a very heavy metal stand. In the eye-piece of the microscope is a scale

moved by a micrometer screw. The scale is carried in a brass mount fitting between the parallel sides of the micrometer box. The micrometer scale mount carries a polished steel plug against which presses the point of the micrometer screw. The carriage is kept pressing against the point of the micrometer screw by a spring-plunger—the spring actuating the plunger being so long that the difference of pressure of the plunger on the micrometer mount at the two extremes of the motion is not great. The scale is graduated so that 100 divisions correspond to the length of a *réseau* line (·5 cm.). The micrometer screw head is graduated into 50 divisions, and one complete turn of the screw moves the scale in the eye-piece through one scale division. The complete reading is obtained by adding the scale and screw readings, the last figure in the latter being read by inspection. Obviously, also, the screw reading has to be doubled before adding it to the scale reading. The screw reading decreases as the scale reading increases, so that the complete reading for any very small and well-defined object should be the same for different positions of the scale.

The scale and eye-piece can be rotated through 90° in order to take readings in all four orientations of the plate.

The eye-piece is a Steinheil 14 mm. equivalent focus A. F achromatic. The lens forming the image is a Zeiss Planar (Series Ia., No. 2).

The photographic plate is firmly clipped to a heavy circular disc, which can be made to rotate about the axis of the microscope. This rotation is perfectly independent of the rotation of the scale. Readings were taken in the order of the orientations 0°, 180°, 90°, 270°. The scale was left in the same position for the first two readings, then rotated through 90° for the last two. Thus, for opposite orientations taken in pairs, the plate alone was reversed, and not the microscope.

Great care was taken to ensure the lighting being adequate and symmetrical. Usually a movable electric lamp was used, placed directly in front of the microscope. When measurements were taken by daylight, the instrument was placed exactly in front of a window.

It is not contended that readings taken with this instrument are necessarily quite as accurate as those taken with the Cambridge machine, but it may be pointed out that any considerable instrumental error affecting the measurements in different orientations in different ways would be shown by the discordances of the means of the results in the different orientations. As a matter of fact, these discordances are very considerably smaller in the case of the last two series than in that of the first, in which the Cambridge instrument was used, though I think this is partly due to the fact that the second star image was smaller than the first.

An instrumental error affecting all the orientations in the same way would have no effect on the results and conclusions given in this paper.

The readings given in the first two series are naturally all

affected by my own personality. In order to eliminate that part of the error due to this cause I obtained readings from 86 independent observers. The star image and the instrument were the same as in the second series.

The experience and training of the observers in this third series varied considerably. Some were well used to investigations on their own account. The majority were students, but all had had some scientific training. I took down the measurements as they were made, and in this way any errors due simply to lack of knowledge of the method and instrument were at once detected.

I used simply the means of the 86 results obtained in the four orientations, as I considered that the accidental errors, which are likely to be larger in this series than in the other two, would be eliminated in taking the means. The fact that the mean of the whole series in four orientations does not differ sensibly from that obtained in the second series shows, I think, that this accidental error has been eliminated.

I did not attempt to correlate results in different orientations in this case, as I considered that individual measurements would probably contain too large an accidental error to give trustworthy results, and also the four measurements in each complete observation would probably be similarly affected owing to their having been taken by the same person. This would almost certainly introduce a marked positive correlation, which might completely mask the correlation (if any) due to the varying orientations.

3. *Nature of the errors involved, and their treatment.*

Any one of the readings given in these three series may be affected by errors of the following kinds:—

1. A purely instrumental error, *i.e.* an error unaffected by the orientation, the observer, the particular star image, or the time and circumstances of the observation. This will be constant.

2. An error of personality, which will probably be partly constant and partly variable in the same orientation, and may also depend upon the orientation.

3. A purely accidental error.

There may also be an error which depends solely on the particular star image chosen, or on the plate. I have not discussed this in the present paper, though I hope to do so later.

Now call the constant parts of the error in each orientation $e_{o,0}$, $e_{o,90}$, $e_{o,180}$, $e_{o,270}$, respectively, and the accidental parts, $e_{a,0}$, $e_{a,90}$, $e_{a,180}$, $e_{a,270}$.

Then, in taking the mean of all the results in each orientation, the accidental error is nearly eliminated, and the difference of each of these means from the mean of the whole series gives the constant error for each orientation of the plate.

I determined these constant errors for each of the three series of observations, and attempted to find the best method of combining measurements in different orientations in order to eliminate them.

In the case of the first two series of measurements, I also found the correlation coefficient for errors in two different orientations. Of course the constant error does not appear in these results, since r, the correlation coefficient, is equal to $\frac{\Sigma(x-\bar{x})(y-\bar{y})}{N\sigma_1\sigma_2}$, where $\bar{x}$, $\bar{y}$ are the mean errors in the two orientations considered. Hence any constant error would be eliminated in the process of finding the coefficient r, and the correlation found is that existing between accidental errors, *i.e.* errors which differ with each observation, but may, nevertheless, be affected by the orientation of the plate.

Taking the readings from the first series, the mean of all the 440 results was found, and each reading subtracted from it. Tables I., II., III., and IV. are correlation tables of the residuals so formed. Tables I. and III. are formed from measurements in opposite orientations of the plate, Tables II. and IV. from measurements in adjacent orientations (0, 90) and (180, 270).

In order to determine whether the correlation coefficient r or the correlation ratio η should be used in this case, the vertical columns were taken from the tables, their means calculated and plotted. A glance at these diagrams showed that, disregarding the extreme means, which were obtained from one or two observations only, the best fitting curve is a straight line with a negative slope in all cases.

TABLE I.

Correlation of errors in 0° *and* 180°.

Orientation 0°.

	−30	−20	−10	0	+10	+20	+30	+40	Totals
−45	...	...	...	...	1	2	...	...	3
−35	...	...	...	1	9	13	4	...	27
−25	1	...	1	4	23	18	2	1	50
−15	...	...	...	5	11	10	2	...	28
− 5	...	...	...	...	...	1	...	...	1
+ 5	...	...	...	...	1	...	...	...	1
Totals	1	0	1	10	45	44	8	1	110

TABLE II.

Correlation of errors in 0° *and* 90°.

Orientation 0°.

Orientation 180.	−30	−20	−10	0	+10	+20	+30	+40	Totals
−40	...	...	...	...	2	...	...	...	2
−30	1	...	...	1	7	9	5	...	23
−20	...	...	1	3	19	21	2	...	46
−10	...	...	...	5	15	11	1	1	33
0	...	...	...	1	4	1	...	...	6
Totals	1	...	1	10	45	44	8	1	110

TABLE III.

Correlation of errors in 90° *and* 270°.

Orientation 90°.

Orientation 270°	−40	−30	−20	−10	0	Totals
−10	...	...	2	1	...	3
0	1	4	14	11	...	30
+10	1	10	14	15	3	43
+20	...	7	11	4	3	25
+30	...	2	5	2	...	9
Totals	2	23	46	33	6	110

TABLE IV.

Correlation of errors in 180° *and* 270°.

Orientation 180°.

Orientation 270°	−45	−35	−25	−15	−5	+5	Totals
−10	...	...	2	1	...	...	3
0	1	5	18	6	...	...	30
+10	...	13	16	13	1	...	43
+20	2	6	9	7	...	1	25
+30	...	3	4	2	...	...	9
Totals	3	27	49	29	1	1	110

TABLE V.

Correlation of errors in 0° *and* 180°.

Orientation 0°.

Orientation 180°	30	−20	−10	0	+10	+20	+30	Totals
−40	...	...	1	1	...	...	...	2
−30	...	1	3	...	...	...	...	4
−20	...	6	5	3	...	...	...	14
−10	1	7	17	5	1	...	...	31
0	...	2	15	4	1	...	...	22
+10	...	...	...	1	1	1	1	4
+20	...	...	...	1	...	...	..	1
+30	...	...	...	...	...	...	...	...
+40	...	...	1	...	...	...	...	1
+50	...	...	...	...	...	...	...	...
+60	...	...	1	...	...	...	..	1
Totals	1	16	43	15	3	1	1	80

TABLE VI.

Correlation of errors in 0° and 90°.

Orientation 0°.

	−30	−20	−10	0	+10	+20	+30	Totals
−30	1	6	8	...	1	...	...	16
−20	...	5	12	4	1	...	...	22
−10	...	4	16	8	1	1	1	31
0	...	1	5	1	...	...	...	7
+10	...	...	2	2	...	...	...	4
Totals	1	16	43	15	3	1	1	80

Hence the use of the correlation coefficient is justified, and it was accordingly calculated for the four tables.

In the case of the readings given in the second series, the correlation coefficient was calculated only for errors in orientations 0° and 180°, and for those in orientations 0° and 90°.

Tables V. and VI. are the correlation tables of these errors.

4. *Results obtained.*

Taking the readings in Series I. first, the mean of the whole was ·3489.

The means in the four orientations were

0°	90°	180°	270°
·3508	·3475	·3468	·3504

Hence $e_{0,0} = +·0019$, $e_{0,90} = -·0014$, $e_{0,180} = -·0021$, $e_{0,270} = +·0015$.

From Series II. (observations on a different star) the results were—

Mean of the whole = ·4104.

Means in the four orientations—

0°	90°	180°	270°
·4107	·4100	·4108	·4102

Hence $e_{0,0} = +·0003$, $e_{0,90} = -·0004$, $e_{0,180} = +·0004$, $e_{0,270} = -·0002$.

From Series III. :—

Mean of the whole = ·41035.

Means in the four orientations—

9°	90°	180°	270°
·4105	·4100	·4099	·4110

$e_{0,0} = -·0001$, $e_{0,90} = -·0004$, $e_{0,180} = -·0005$, $e_{0,270} = +·0006$.

Secondly, the results given by the correlation coefficients were as follows:—

Table I.	$r = -\ \cdot16 \pm \cdot06$	From first series of observations.
Table II.	$r = -\ \cdot18 \pm \cdot06$	
Table III.	$r = -\ \cdot20 \pm \cdot06$	
Table IV.	$r = -\ \cdot04 \pm \cdot07$	
Table V.	$r = +\cdot225 \pm \cdot07$	From second series of observations.
Table VI.	$r = +\cdot265 \pm \cdot07$	

5. *Discussion of results.*

In the case of the readings in Series I. which were taken with the Cambridge plate-measurer, the results show that, as far as the constant error is concerned, the best elimination is obtained by combining readings in opposite orientations of the plate, and little or nothing is gained by taking four orientations. The mean result of grouping readings in 0° and 180° is ·3488, while readings in 90° and 270° give ·3489. The agreement with the mean of the whole is remarkable.

On the other hand, if we take the means from Series II., the best compensation of the constant error is undoubtedly given by grouping adjacent orientations rather than opposite ones. Any pair of adjacent orientations, when taken together, will give a mean result very close to that obtained from readings in all four orientations.

The results from Series III. (independent observers) do not quite bear this out. In this case the error of personality is presumably eliminated, and the constant error remaining is due to the instrument, the method of measurement, and the star image itself.

Now in this case the mean of the whole series does not differ sensibly from that given by my own observations in Series II., but the means in the separate orientations, with the exception of orientation 90°, are all affected by personality.

The best compensation of the constant error is given here by combining 180° with 270°, or 90° with 270°.

The result of combining measurements in 0° with those in 90°, or those in 0° with those in 180°, is not very far from the mean of the whole; but, on the other hand, 0° taken with 270°, or 90° with 180°, gives no compensation whatever. It seems difficult, then, in this case to determine any definite rule for the choice of orientations; and, as far as the constant error is concerned, it almost seems here that the four orientations advocated by Loewy are necessary. The result given by four orientations is, in my own case, practically unaffected by personality.

I intend shortly to make further series of measurements on different star images. If these series should give similar results, a grouping of 0° with 90°, or 180° with 270°, might perhaps be selected as giving good compensation of error for this particular instrument.

It is also, of course, true that the error due to personality depends very largely on the star image itself, and it is quite likely that further work on different star images may give quite different results, even though the same instrument be used. Quite possibly it may be found that for different star images different groupings of orientations give the best compensation of error; and if this be so, the case for Loewy's four orientations would seem to be a strong one. It is impossible, however, to say anything definite on this point until considerably more work has been done, but I hope to be able to throw more light upon it later.

The effect of my personality on the readings in different orientations is rather curious. The differences of the means in the last series and in my own are—

0°	90°	180°	270°
+·0002	0	−·0009	+·0008

It is a little difficult to account for the much more marked effect in the last two orientations. It is quite probably due to a physiological cause, and may perhaps be explained by the rotation of the circular disc to which the plate is clamped. This does, as a matter of fact, produce a slight shift in the position of the star image on the field of view, which might give rise to errors due to astigmatism.

This result of personality was not apparent in the Cambridge machine, since there the combinations of 0° with 180° and 90° with 270° gave such very close results, and it is noteworthy that in this case the plate itself was not moved, the differences of orientation being obtained by means of the revolving prism in the eye-piece. It seems, then, that the error of personality is decidedly affected by the particular instrument used, and may therefore, in some sense, be regarded as an instrumental error.

Next, as regards the accidental errors, the values of the correlation coefficient r, obtained from Tables I., II., III., and IV., show that there is a certain tendency for a small (or negative) error in one orientation to be associated with a large (or positive) error in another, but this applies equally to the groupings in opposite and adjacent orientations. The value of r from Table IV. is certainly smaller than those from the other three tables; but, considering the rough grouping, the difference is not marked enough for any significance to be attached to it.

The extreme smallness of the correlation coefficients may be due to two causes:—

(1) The variable error may be divided into two parts, one of which, though varying from one complete observation to another, is yet affected by the orientation, and the other, purely accidental. Calling the parts of the error dependent on the orientation, $e'_{a,\,0}$, $e'_{a,\,90}$, $e'_{a,\,180}$, $e'_{a,\,270}$, and the parts independent of it, $e''_{a,\,0}$, $e''_{a,\,90}$, $e''_{a,\,180}$, $e''_{a,\,270}$,

we have

$$e_{a0} = e'_{a0} + e''_{a0}$$
$$e_{a90} = e'_{a90} + e''_{a90}$$

where e_{a0}, e_{a90} are the complete accidental errors in orientations 0° and 90°.

Now, in forming the correlation coefficient we calculate

$$\Sigma e_{a0} e_{a90} = \Sigma e'_{a0} e'_{a90} + \Sigma e'_{a0} e''_{a90} + \Sigma e'_{a90} e''_{a0} + \Sigma e''_{a0} e''_{a90}.$$

Since e''_a is purely accidental, we get

$$\Sigma e_{a0} e_{a90} = \Sigma e'_{a0} e'_{a90},$$

or the r can only be affected by the magnitude of the standard deviations.

Now $\sigma^2_{a0} = \sigma'^2_{a0} + \sigma''^2_{a0}$

$\sigma^2_{a90} = \sigma'^2_{a90} + \sigma'^2_{a90}$.

And $r_{0,\,90} = \dfrac{\Sigma e_{a0},\, e_{a90}}{N \sigma_{a0} \sigma_{a90}}$.

Hence the presence of this *purely* accidental error will make the correlation coefficient smaller numerically than it would have been had the whole accidental error been dependent on the orientation.

(2) There is probably a certain amount of positive correlation existing between the purely accidental errors. Any cause which produced a large error in a measurement on a particular day, for example, would quite probably affect all the errors for that day in the same way.

This would still further mask the effect of the negative correlation between the parts of the accidental error dependent on the orientation.

In the second series of observations the work extended over several months, and the circumstances in which the reading were taken varied considerably.

It was to be expected, then, that the positive correlation of the purely accidental errors would be more marked in this case, and we find that, as a matter fact, r is positive for both the correlation tables.

This does not necessarily mean that there is no compensation of the accidental errors when readings in two orientations are combined; it may merely show that the positive correlation referred to above is strong enough to mask the effect of the compensation of error indicated by the negative correlations obtained in the first series.

Now the fact that the correlation coefficients obtained from the Cambridge series are all practically equal shows that, as far as the accidental error is concerned, no advantage is gained by com-

bining any particular pair of orientations. Hence the whole of the compensation obtained when opposite orientations are combined is due either to the instrument, or to the constant part of the error of personality, or most probably to both. If, then, the complete constant error is not considerably greater than the accidental error, the advantage gained by combining opposite orientations is not so marked as it would appear from a consideration of the means alone.

As a matter of fact, the test given below for combined constant and accidental error shows that in the Cambridge series the advantage gained by taking opposite orientations is not much greater than that obtained by taking adjacent ones.

6. *Combined constant and accidental error.*

I next tried to see if it were possible to determine the best combination of orientations to give compensation of the total error, variable and constant.

If the whole error is best eliminated by combining readings in 0° and 180°, for instance, rather than those in 0° and 90°, then

$$\Sigma\left(\frac{e_{o,\,0}+e_{o,\,180}}{2}+\frac{e_{a,\,0}+e_{a,\,180}}{2}\right)^2 \text{ should be}$$

$$\text{less than } \Sigma\left(\frac{e_{o,\,0}+e_{o,\,90}}{2}+\frac{e_{a,\,0}+e_{a,\,90}}{2}\right)^2$$

$$\text{or } \Sigma\left(\frac{e_{o,\,0}+e_{o,\,180}}{2}\right)^2+\frac{\sigma^2_{a,\,0}+\sigma^2_{a,\,180}+2r_{a0,\,a180}\,\sigma_{a.\,0}\,\sigma_{a,\,180}}{4}$$

$$< \Sigma\left(\frac{e_{o,\,0}+e_{o,\,90}}{4}\right)^2+\frac{\sigma^2_{a,\,0}+\sigma^2_{a,\,90}+2r_{a0,\,a90}\,\sigma_{a0}\,\sigma_{a90}}{4}.$$

For the readings in Series I. (taken with the Cambridge machine) the value of the first expression (0,180) is 30·8, that of the second (0,90) is 36·4. Hence opposite orientations appear to have a certain advantage over adjacent ones in this case. For the readings in Series II. the corresponding values are 93·6 and 58·4. Here the adjacent orientations give very considerably better results. The effect of the accidental error is much more marked in the second case, while the constant error is much smaller than in the Cambridge series. The large constant error in the first set points, I think, to a decided error of the instrument, which is, however, practically eliminated in taking the mean of readings in opposite orientations.

In each of these cases, then, the combination of orientations which best eliminates the constant error gives also the best compensation of the total error, but the advantage of taking one grouping rather than another is much more markedly shown in the second case.

Taking, now, the third series of observations, I tried to find

whether the four orientations advocated by M. Loewy really gave better results than two.

Obviously in this case the error of personality forms part of the accidental error, since it will differ with each observer, but in any one complete observation it will probably be affected by the orientation.

Calling the complete accidental errors $E_{a,0}$, $E_{a,90}$, $E_{a,180}$, $E_{a,270}$, and using the same notation as before for the constant errors, I compared the values of

$$\Sigma\left(\frac{E_{a,0}+E_{a,90}+E_{a,180}+E_{a,270}}{4}\right)^2 \quad \text{and} \quad \Sigma\left(\frac{e_{o,0}+e_{o,90}+E_{a,0}+E_{a,90}}{2}\right)^2.$$

The value of the first was 451·75 while the second was 770·72.

Now if we suppose all the errors $E_{a,0}$, $e_{o,0}$ (0, 90, 180, 270) to be entirely uncorrelated and of the same average magnitude in each case, then we should have the first quantity $\frac{N\sigma^2}{16}$ and the second $\frac{N}{2}\frac{\sigma^2}{4}$, σ being the mean square or standard deviation and N the total number of observations.

Hence, quite independently of any correlation, we should expect the first quantity to be half the second.

The fact, then, that the first value is actually more than half the second tells apparently against four orientations, since the advantage gained by taking four such systematic measurements is less than that obtained by taking four random measures.

7. *Conclusions.*

The conclusions I have reached so far are as follows:—

1. There is a very slight correlation between the accidental errors in different orientations. This correlation is, however, the same for all groupings of orientations which have been considered, and therefore, as far as the accidental error is concerned, no advantage is gained by combining any two particular orientations.

2. In the case of the Cambridge machine, the combination of readings in opposite orientations of the plate practically eliminates the constant error, and this combination also, as would be expected, gives the best results for the combined constant and accidental error. The advantage of opposite over adjacent orientations is not, however, very marked.

3. With my own instrument there is evidence that a combination of readings in adjacent orientations gives very considerably better results than those obtained by grouping 0 with 180 or 90 with 270.

As far as my own readings are concerned, adjacent orientations give results very close to the mean of the whole, but this is not fully borne out by the results of the independent observers.

It must be remembered that with both series measurements were made on a single star, and it is probable that different results would be obtained with different star images.

4. The constant error is affected by the star image used, and may be small in comparison with the accidental error.

Finally, I am led to the conclusion that it is dangerous to assume that any particular association of two orientations will give the best results. The best groupings to take differ with different instruments and different observers, and probably with different star images; and unless a preliminary series of measurements can be made, it certainly seems that M. Loewy's four orientations are necessary if the final result is to be trustworthy.

In conclusion, I have to express my thanks to the Royal Society for aid in the expenses of the work, to the Director of the Cambridge Observatory and Mr. Hinks for many facilities afforded me at Cambridge, and to the students and staff of the Goldsmiths' College, and others, for the time and trouble expended in making the series of independent measurements for me.

I have also to thank Messrs. Adam Hilger for their care and thought in the construction of the plate measurer.

The Constitution of the Solar Corona. I.: Protofluorine.

By J. W. Nicholson, M.A., D.Sc.

In a previous paper the nebular spectrum was discussed from the point of view of a definite theory of atomic constitution, and it was shown that there is reason for the belief that most of the lines of this spectrum may be due to a substance which was called nebulium, a more elementary form of matter than any yet discovered on the Earth. The theory in question involves also the existence of other simple forms, and an application of the method of the last paper has indicated that the free vibrations of one of these forms, which was called protofluorine when the theory was first introduced, do not appear to be represented in the nebular spectrum. In these circumstances it is natural, in the search for a verification of its existence, to turn our attention to the coronal spectrum, where matter is also believed to give evidence of its most elementary forms. The present paper takes up the examination of this spectrum on the previous lines, and considerable evidence is adduced to show that this substance is capable of giving rise to a large number of the coronal lines.

Since the paper is a natural continuation of the former, it is not necessary to repeat the analysis. Protofluorine differs from nebulium only in the fact that its central positive charge is $5e$, whereas that of nebulium is only $4e$, the electronic charge being e. When its atom is electrically neutral it contains 5 electrons

rotating at equal distances in a circular orbit round the positive nucleus.

It is evident that the analysis of the previous paper will apply also to this case if the quantity ν of that paper now has the value 5 instead of 4, and we may proceed at once to the numerical calculations.

The period equations for the transverse vibrations of an atom containing a single ring of electrons are of the form

$$q^2 = e^2\{\nu + (P_\kappa - P_o)/8\}/ma^3 \qquad \kappa = 0,\ 1,\ 2,\ \ldots\ .$$

and the condition for steady motion is

$$\omega^2 = e^2(\nu - S_n/4)/ma^3$$

where ω is the angular velocity, m and a are the mass and radius of an electron, and

$$S_n = \Sigma_1^{n-1} \operatorname{cosec} \pi/n, \qquad P_\kappa = \Sigma_{s=1}^{n-1} \cos 2\kappa s\pi/n.\ \operatorname{cosec}^3 s\pi/n.$$

The values of P_κ quoted in the preceding paper were—

$n = 2$

$P_0 = 1, \qquad P_1 = -1.$

$n = 3$

$P_0 = 16/3\sqrt{3}, \qquad P_1 = -8/3\sqrt{3}, \qquad P_2 = -8/3\sqrt{3}.$

$n = 4$

$P_0 = 1 + 4\sqrt{2}, \qquad P_1 = -1, \qquad P_2 = 1 - 4\sqrt{2}.$

$n = 5$

$P_0 = 12{\cdot}1732, \qquad P_1 = 1{\cdot}1609, \qquad P_2 = -7{\cdot}249.$

$n = 6$

$P_0 = 17 + 16/3\sqrt{3}, \quad P_1 = 7 - 8/3\sqrt{3}, \quad P_2 = -7 - 8/3\sqrt{3}, \quad P_3 = -17 + 16/3\sqrt{3}.$

Periods of the neutral atom.—The neutral atom is defined by $= 5$, $n = 5$, so that

$$q^2 = e^2(5,\ 3{\cdot}62346,\ 2{\cdot}5722,\ 3{\cdot}62346,\ 5)/ma^3$$

quoting the values of P_κ in the succession $\kappa = 0,\ 1,\ 2,\ \ldots\ .$
But

$$\omega^2 = 3{\cdot}62346e^2/ma^3,$$

and therefore

$$q^2/\omega^2 = (1{\cdot}37988,\ 1,\ 0{\cdot}70987,\ 1,\ 1{\cdot}37988)$$

or

$$q/\omega = \left\{ \begin{matrix} 1{\cdot}1747, & 1, & 0{\cdot}84253, & 1, & 1{\cdot}1747 \\ \kappa = 0 & \kappa = 1 & \kappa = 2 & \kappa = 3 & \kappa = 4 \end{matrix} \right\}$$

The frequencies relatively to an observer, given by $q \pm \kappa\omega$, are therefore given by

$$q/\omega = \left\{ \begin{matrix} 1{\cdot}1747, & 1{\cdot}15747 \\ \kappa = 0 & \kappa = -2 \end{matrix} \right\}$$

retaining only those which can fall in the visible spectrum together, the vibration $\kappa = 0$ being supposed to be a strong spectral line as in the case of nebulium. We may add the principal vibration in the plane of the ring, given by $q = \omega$, and thus obtain the system

$$q/\omega = \{1,\ 1{\cdot}1747,\ 1{\cdot}15747\}.$$

The third of the set, corresponding to $\kappa = -2$, cannot be expected to be absent if the others are very strong, for the coronal vibrations are much stronger than those of the nebulæ, in which a line corresponding to $\kappa = -2$ for a neutral atom cannot be expected to appear.

The accompanying wave-lengths are

$$\lambda = 2\pi c\{1,\ 0{\cdot}85128,\ 0{\cdot}86396\}/\omega.$$

We shall take the coronal lines given by Dyson * as the standard of comparison. They are the results of the measurements made in 1900, 1901, and 1905. The three strongest lines are as follows:—

$\lambda = 5303{\cdot}1,$	Intensity 20.
$\lambda = 3387{\cdot}9,$	Intensity 12.
$\lambda = 3454,$	Intensity 9.

The second and third were observed together in 1901 in Sumatra. There is also a fairly strong line, of intensity 3, at $\lambda = 3987{\cdot}1$, which was observed in each year. It is, of course, among the strong lines that we should expect to find the vibrations of the neutral atom, as in the case of nebulium, if the substance now under consideration is actually present in the corona.

Let us identify the first line of the present set with the line $\lambda 3987{\cdot}1$. Then the second should have a wave-length $3987{\cdot}1 \times {\cdot}85128$ or 3394. This differs by only 6 tenth-metres from the strong line $3387{\cdot}9$. Moreover, the third should have a wave-length $3987{\cdot}1 \times {\cdot}86396$ or 3445, which differs from the line $\lambda 3454$ by 9 tenth-metres. The agreement is very good, and we may therefore assign the lines

$$\lambda 3987{\cdot}1, \quad \lambda 3387{\cdot}9, \quad \lambda 3454$$

to the neutral atom of protofluorine, if the error is found to be within the proper limits. The next section gives an investigation

* *Phil. Trans.*, 1906, A, vol. 206, p. 451.

of the atom on this assumption, and it is found that the error is not too large, although larger than in the case of the neutral atom of nebulium. The source of the increase can lie in the fact that the velocity of the electrons in the protofluorine atom, when neutral, is nearly twice as large, so that the magnetic effects of the motion are more important.

Characteristics of the neutral atom.—On the basis suggested above, a line of wave-length $2\pi c/\omega$ was identified as the coronal line $\lambda 3987{\cdot}1$, so that

$$2\pi c/\omega = 3{\cdot}987.10^{-5}$$

when λ is expressed in centimetres. Thus the angular velocity in the atom is given by

$$\omega/c = 1{\cdot}576.10^{5}$$

as against $1{\cdot}26.10^{5}$ for nebulium. The equation of steady motion is

$$ma^3\omega^2/e^2 = 3{\cdot}6235.$$

With $e = 3{\cdot}4.10^{-10}$, $e/mc = 1{\cdot}88.10^{7}$, according to the accepted estimates

$$\omega^2 a^3 = 6{\cdot}948.10^{8},$$

whence

$$a = 5{\cdot}802.10^{-8},$$

so that the atom is somewhat larger than that of nebulium.

If v is the velocity of an electron,

$$v/c = a\omega/c = {\cdot}00917$$

and v is $2{\cdot}1$ times greater than the corresponding velocity in nebulium. The error in the investigation may therefore be about twice or four times as great, according as v/c or its square is the most significant order neglected. The average error in the case of nebulium was about 3 tenth-metres, so that errors up to 7 or 8, or perhaps 12 tenth-metres, are admissible at present.

The identifications suggested for the neutral atom in the previous section thus receive confirmation.

The system of coronal lines.—The following table of coronal lines is that given by Dyson,* mean values being assigned to a wave-length when possible. The total number of lines is 24, within a range of wave-length of 2175 tenth-metres. It is significant that the two very strong lines, of intensities 9 and 12, have only appeared once, and simultaneously, perhaps at an instant in which the corona was composed to an unusual extent of neutral atoms.

Their companion, 3987·1, has appeared on all occasions, so that it may not be a single line, and may have another partial origin. This conclusion receives some justification later.

* *Loc. cit.*

1900.	1901.	1905.	Mean.	Intensity.
...	...	5535·8	5535·8	2
...	5304	5303·1	5303·1	20
...	...	5117·7	5117·7	2
5073	...	...	5073	1
4779	...	...	4779	1
4725 ?	...	...	4725	...
4722 ?	...	...	4722	...
4586·3	...	...	4586	4
4566·5	4565	...	4566	6
4400	...	...	4400	1
4358·8	...	...	4359	4
4311·3	...	...	4311	2
4230·6	4230·9	4231·1	4231·0	5
4130	...	...	4130	...
...	...	4087·4	4087	...
3987·2	3987	3987·1	3987·1	3
...	3891·2	...	3891	...
3800·8	3801·1	3800·8	3800·9	3
3642·9	...	3642·0	3642·5	2
...	3505 ?	...	3505 ?	...
3461·3	...	...	3461	1
...	3454	...	3454	9
...	3387·9	...	3387·9	12
...	3361 ?	...	3361 ?	...

Protofluorine with a single positive charge.—When the atom loses an electron, we have $\nu = 5$, $n = 4$. In this case we should expect at once that, if the lines are represented in the spectrum, it should be possible to calculate them with an error of about 3 to 3·5 tenth-metres only, in accordance with the error for four electrons in the case of nebulium. This expectation will be realised. The period equations become—

$$q^2 = e^2(5,\ 4{\cdot}04289,\ 2{\cdot}58579,\ 4{\cdot}04289,\ 5)/ma^3$$

with the condition of steady motion—

$$\omega^2 = 4{\cdot}04289\ e^2/ma^3$$

so that

$$q^2/\omega^2 = (1{\cdot}2367,\quad 1,\quad 0{\cdot}88693,\quad 1,\quad 1{\cdot}2367)$$

or

$$q/\omega = \begin{pmatrix} 1{\cdot}11207, & 1, & 0{\cdot}94177, & 1, & 1{\cdot}11207 \\ \kappa=0 & \kappa=1 & \kappa=2 & \kappa=3 & \kappa=4 \end{pmatrix}.$$

Adding the period given by $q=\omega$, and retaining only the vibrations which can be in the visible spectrum together, the relative frequencies for a stationary observer are given by

$$q/\omega = \left\{ \underset{\kappa=0}{1},\ \underset{\kappa=0}{1{\cdot}11207},\ \underset{\kappa=-2}{1{\cdot}05823} \right\}$$

with wave-lengths—

$$\lambda = 2\pi c(1,\ 0{\cdot}89923,\ 0{\cdot}94497)/\omega,$$

where the last is probably invisible, as corresponding to $\kappa=-2$, a class of vibrations not of great intensity for an atom positively charged, as was the case with nebulium.

Take the first of these lines as the fairly strong coronal line λ4231 of intensity 5, observed in each year. Then the second should have a wave-length 3804·5, differing from the strong line λ3800·9 by only 3·6 tenth-metres. This second line is always present with the first, and has an intensity 3 in Dyson's Table.

The third line should have a wave-length 3998, differing from the line λ3987·1 by 11 tenth-metres. This error is too large to be accepted in the present case. We conclude that the line, as expected, has not a visible intensity, and that the rough coincidence is accidental.

We note that the error in the other case is only 3·6 tenth-metres, of the amount suggested at the beginning of the present section. This suggestion may be fortified by a calculation of the characteristics of the ionised atom. For the above identification gives the angular velocity of the electrons as $\omega = 1{\cdot}485\mathrm{C}.10^{-5}$. The radius of the atom becomes $a = 3{\cdot}393.10^{-8}$, so that it is smaller than the neutral atom. The velocity of an electron is in a ratio ·00504 to that of light, and is slightly greater than in electrically neutral nebulium. The comparison of the errors, 3·6 and 3·2 respectively, is exactly in accord with this.

We conclude, therefore, that the identification of the lines λ4231 and λ3800·9 is justified.

Protofluorine with a double positive charge.—When the atom has lost two electrons, $\nu=5$, $n=3$, and

$$ma^3q^2 = e^2\left(\underset{\kappa=0}{5},\ \underset{\kappa=1}{5-1/\sqrt{3}},\ \underset{\kappa=2}{5-1/\sqrt{3}}\right)$$

$$ma^3\omega^2 = e^2(5-1/\sqrt{3})$$

and therefore

$$q/\omega = (1{\cdot}0633,\ 1,\ 1).$$

Forming the frequencies relatively to an observer, and adding the vibration $q=\omega$, those in the visible spectrum become

$$q/\omega = (1,\ 1{\cdot}0633)$$

with wave-lengths

$$\lambda = 2\pi c(1,\ 0{\cdot}94047)/\omega.$$

There are two lines with the proper ratio in the coronal spectrum, both of which were observed in 1900, but not in later eclipse expeditions. They were then fairly strong, the intensities being given as 4 and 2 respectively.

Identifying the first line with λ4586, the second becomes 4313, exhibiting only 2 tenth-metres difference from the observed line λ4311. The error is smaller than before, as would be expected from the smaller number of moving electrons.

We may note also the steady decrease in the intensities of the sets of lines which has accompanied a decrease in the number of electrons in the atom. This again would be expected, for it must be increasingly difficult to detach more electrons from the atom, so that smaller proportions of the total number of atoms would be in the successive stages of detachment at any given time.

A calculation of the characteristics of the present atom shows that its radius is $3{\cdot}689.10^{-8}$ cms., the angular velocity is $1{\cdot}370.10^{5}$C, and the ratio of the velocity of an electron to that of light is ·00504, or nearly the same as in the previous case. But a decrease in the number of rotating electrons must also cause a decrease in the probable error of the calculated lines, which is only 2 tenth-metres in the present case, or 1 tenth-metre for each line.

Protofluorine with a triple positive charge.—In this state, the atom only retains two electrons, the minimum number which does not lead to excessive radiation and perhaps atomic destruction. It is difficult to make a terrestrial atom lose so large a proportion of its electrons, and such a system in the corona could only therefore be expected to give very weak lines. But the electrical agencies in the corona are very strong, and it is probable that ionisation to this degree is produced to some extent. The error in the estimate of wave-lengths should not, in the present case, exceed about 3 tenth-metres. It is not necessarily less than before, on account of greater radiation and consequent æthereal reaction on the electrons, which, as Sir J. J. Thomson has shown, becomes important with only two electrons. A rough calculation indicates that about a tenth-metre or two of error should be thus introduced. The case is defined by $\nu = 5$, $n = 2$, and

$$q^2 = e^2(5,\ 4{\cdot}75,\ 5)/ma^3$$
$$\omega^2 = 19e^2/4ma^3$$

so that

$$q/\omega = \left\{ \begin{matrix} 1{\cdot}02594, & 1, & 1{\cdot}02594 \\ \kappa=0 & \kappa=1 & \kappa=2 \end{matrix} \right\}$$

and with the vibration in the plane, given by $q = \omega$, an observer may perceive vibrations

$$q/\omega = (1,\ 1{\cdot}02594)$$

in the visible spectrum, of wave-lengths

$$\lambda = 2\pi c(1,\ 0{\cdot}97472)/\omega.$$

Identify the first with the weak line λ4087·4, observed only in 1905, of tabular intensity zero. Then the second has a wave-length 3984·2, within 2·9 tenth-metres of the line λ3987·1, which, being a stronger line due to another atom, should completely mask it. A suspicion that this line may be double has already been expressed on other grounds. The error is of the amount expected, and it is probable that the present interpretation is valid.

On this basis, the radius of the present atom becomes $3{\cdot}499.10^{-8}$ cms., the angular velocity $1{\cdot}537.10^{5}$C, and the velocity of an electron ·00538C.

With the exception of the other vibrations in the plane of the atom, positively charged and neutral atoms have now been examined in detail, and they have accounted for 8 lines. We proceed to a consideration of negatively charged atoms.

Protofluorine with a single negative charge.—The spectral lines to which this system gives rise should be fairly strong, for an appreciable number of the atoms should take up one electron at least. The case is $\nu=5$, $n=6$, and we find—

$$q^2/\omega^2=\begin{pmatrix} 1{\cdot}57604, & 1, & 0{\cdot}44838, & 0{\cdot}2364 & 0{\cdot}44838, & 1 \\ \kappa=0 & \kappa=1 & \kappa=2 & \kappa=3 & \kappa=4 & \kappa=5 \end{pmatrix}$$

or

$$q/\omega=(1{\cdot}2554,\ 1,\ 0{\cdot}66961,\ 0{\cdot}4862,\ 0{\cdot}66961,\ 1).$$

The frequencies in the visible spectrum, manifested to a stationary observer, are given by

$$q/\omega=(1,\ 1{\cdot}2554,\ 1{\cdot}33039)$$

where the first is the principal vibration in the plane. Their wave-lengths are

$$\lambda=2\pi c\{1,\ 0{\cdot}79656,\ 0{\cdot}75166\}/\omega,$$

and the last corresponds to $\kappa=-2$, and may not be strong.

In this case two possible identifications present themselves, but a choice may be made without difficulty. The alternatives offer an interesting case of an accidental correspondence, the second of the kind met with in the course of the work. In neither case is the third line represented in the spectrum.

The first alternatives consist of an identification of the first line with λ4566, a strong line in the corona, of intensity 6. This gives the second a wave-length 3637·2, differing from the coronal line λ3642 by 4·8 tenth-metres. The third line then has a wave-length 3432, and is not represented in the spectrum. This fact is not unexpected in any case.

In the second alternative, the first line is identified with λ5303·1, of intensity 20, the strongest line in the spectrum. The next line then has a wave-length 4224·2, differing by 6·8 from λ4231. The third is then almost exactly coincident with λ3987·1. But

this exactness is a proof of mere chance, for there must be an error of about 4 tenth-metres in the reckoning, so that λ3987·1 cannot be this line, and we conclude that it is not represented. Moreover, both the lines λ4231 and λ3987·1 have been interpreted otherwise in modes for which the evidence is very precise. Finally, the error is larger in this case, being 6·8 as against 4·8, and, moreover, the permissible error is decidedly smaller, as may be seen if the characteristics of the atom are worked out in correspondence with each case. On all lines of reasoning, therefore, the second alternative must be rejected, and we conclude that the first is to be accepted, so that the present atom gives rise to the lines

$$\lambda 4566,\ \text{Int.} = 6 \qquad \lambda 3642,\ \text{Int.} = 2.$$

The intensities are in accord with expectation. In the other alternative, the strength of the primary line would be difficult to explain.

A determination of the characteristics of the atom in the selected case gives

$$\omega = 1{\cdot}376.10^{5}\text{C}, \qquad a = 3{\cdot}293.10^{-8}, \qquad a\omega/c = {\cdot}00453,$$

from which we see that the error has the proper value.

It is evident, therefore, that fortuitous equalities of ratios do occur, but that there is little difficulty in detecting them by the present methods.

Protofluorine with a double negative charge.—For this and other cases we shall require further values of the functions P_κ. The following results have been obtained:—

$n = 7$

$$S_7 = 7{\cdot}62406, \qquad P_0 - P_1 = 15{\cdot}2481,$$
$$P_0 - P_2 = 49{\cdot}9772, \qquad P_0 - P_3 = 85{\cdot}8528,$$

$n = 8$

$$S_8 = 11{\cdot}21946, \qquad P_0 - P_1 = 22{\cdot}4389, \qquad P_0 - P_2 = 125{\cdot}1738.$$

For the case of seven electrons, therefore,

$$\omega^2 = 3{\cdot}09398e^2/ma^3$$
$$q^2 = e^2 \left\{ \begin{matrix} 5, & 3{\cdot}09398 \\ \kappa = 0 & \kappa = 1 \end{matrix} \right\} / ma^3$$

and the values of q^2 for κ greater than unity become negative, except the last two, which cannot give vibrations in the spectrum.

Some of the degrees of freedom are therefore unstable, but this instability is not necessarily absolute. It may be determined by the assumed smallness of the displacement throughout the motion. If absolute, it may mean that an electron becomes detached at a certain stage, but as soon as the new atom has been adjusted, this electron could return. These unstable atoms must be supposed to

be formed and dissociated alternately, and it is difficult to believe that they cannot at any time exist in sufficient numbers to give definite spectrum lines. Instability of a single atom has been urged against the model of Nagaoka, but in a mass of gas, for example, in which electrons are free to move from unstable to stable atoms, the criticism does not seem valid against the actual existence of model atoms of this type. There are many physical phenomena which can readily receive an explanation if the *individual* atoms of a body cannot stably hold all their electrons. For example, the unstable vibrations of the case $n=7$ above are in the ultra violet, and this fact may have bearing upon the effect of ultra violet light in liberating electrons from bodies.

Without pursuing this inquiry further, we may expect therefore to find spectrum lines corresponding to the present atom with seven electrons. The unstable vibrations, as stated, are found to be in the ultra violet, and need not be considered further in this paper. But the stable vibration $\kappa=1$, and the vibration $q=\omega$ in the plane, lead ultimately to a pair of wave-lengths, manifested to an observer, of the form

$$\lambda = 2\pi c(1,\ 0{\cdot}7866)/\omega.$$

Identify the first line with $\lambda 5535{\cdot}8$, observed in 1905, with intensity 2. Then the second should be $4354{\cdot}6$, differing from $\lambda 4359$, of intensity 4, observed in 1900 (and perhaps in 1905 when the plates were fogged in this region) by $4{\cdot}4$ tenth-metres.

The characteristics of this atom are found to be

$$a = 3713.10^{-8}\text{ cms.}, \qquad \omega/c = 1{\cdot}135.10^{5},$$
$$a\omega/c = {\cdot}00429$$

so that the error is of the expected amount, and we may identify the lines

$$\lambda 5535{\cdot}8 \quad \text{and} \quad \lambda 4359$$

as due to the present atomic system.

Protofluorine with eight electrons.—Consider finally the same system with eight electrons. Most of the vibrations are unstable modes, and the system is less permanent than in the last case, but nevertheless may persist for a sufficient time, and be re-formed in sufficient quantity, to give visible lines. We find

$$\omega^2 = 2{\cdot}19513e^2/ma^3$$
$$q^2 = e^2\{5,\ 2{\cdot}19513\}/ma^3$$

selecting the stable vibrations which can fall in the visible spectrum. Thus

$$q/\omega = \{1{\cdot}50923, \quad 1\}$$
$$\kappa=0 \qquad \kappa=1$$

and with the vibration $q = \omega$ in the plane, the possible wavelengths for a stationary observer are

$$\lambda = 2\pi c\{1, 0{\cdot}66252\}/\omega$$

which are so far apart that they can only be in the spectrum together if they are at opposite ends.

If the first is λ5073, a coronal line of intensity unity in Dyson's table, the second becomes 3361·3, which is very close to the line given, with doubt, at λ3361, which was observed faintly in 1905.

As this line is uncertain, the coincidence is perhaps not so exact, as of course it must not be on the present theory.

We may therefore, but more tentatively in this case, identify the two lines in this way. It is possible, however, that one line is outside the spectrum, so that the other cannot be identified by the method of ratios.

Since there are two stable vibrations for $n = 9$ also, and for $n = 10$, there may be two more very weak lines in the coronal spectrum, but their identification cannot be made, as their companions are outside the spectrum.

The lines which have been ascribed to protofluorine are collected in the following table, in which the charge carried by the vibrating ion is specified. Probably two other weak lines should be added to the list.

There is no necessity to give an estimate based on the theory of probability in this case, for the result would evidently be similar to that obtained in the case of nebulium. Moreover, the analysis has been more rigorous, in that a greater amount of discussion of intensity and of probable error has been given.

Coronal Line.		Suggested Origin.	Intensity.
5535·8	Pf,	$-2e$	2
5073	Pf,	$-3e$ (?)	1
4586	Pf,	$+2e$	4
4566	Pf,	$-e$	6
4359	Pf,	$-2e$	4
4311	Pf,	$+2e$	2
4231	Pf,	$+e$	5
4087	Pf,	$+3e$ (?)	...
3987·1	Pf,	neutral atom, ($+3e$?)	3
3800·9	Pf,	$+e$	3
3642·5	Pf,	$-e$	2
3454	Pf,	neutral atom	9
3387·9	Pf,	neutral atom	12
3361 (?)	Pf,	$-3e$ (?)	...

With regard to the other lines, including the strongest line

λ5303·1, an opinion will not be expressed at this point, but it is probable that a similar origin may be prescribed. It has been shown, however, that the dynamical system proposed can give a close correspondence with the 14 coronal lines in the table, which include, with one exception, all the stronger lines.

Trinity College, Cambridge:
1911 *November* 20.

Note on the Visual Observation of certain Stellar Gaseous Nebulæ. By the Rev. T. E. Espin, M.A.

During the autumn certain of the gaseous stellar nebulæ, detected either visually or photographically with the prism, have been observed with the 17¼-inch. As they are generally faint, and undistinguishable from stars with a low power, they have been in each case identified by placing a direct-vision prism before the eye-piece. Where possible, the nebula has been connected with a neighbouring star by means of the micrometer.

N.G.C. 6766 was searched for on Sept. 19th and Oct. 10th, but as it could not be found, and as the place was identical with N.G.C. 6884, save in the hour of R.A., a letter was addressed to Professor Pickering concerning the object, and he informs me that the position was erroneously given in *A.N.*, 105, and that the object is really identical with N.G.C. 6884. The nebula was detected on May 8th, 1883, by him, and by Dr. Copeland, Sept. 20th, 1884, *M.N.*, xlv. 2, p. 91.

N.G.C. 6833, R.A. $19^h\ 46^m\ 55^s$, Decl. + 48° 42′·5 (Pickering) (1900).

1911 Sept. 24th.—Magnitude 12·0. No disc or nebulosity apparent, and no bright star convenient to be connected by the micrometer.

Oct. 10th.—Magnitude 11·5, suspected some nebulosity.

Oct. 28th.—Magnitude 12·0, sharp and well-defined, no nebulosity seen. It is the N. star of a triangle in the field, the two stars forming the base being 13·5. The *f* side of the triangle is marked by two 13·5 mag. stars and the nebula, all in a nearly straight line. The faint stars too small for accurate measurement.

N.G.C. 6884, R.A. $20^h\ 7^m\ 15^s$, Decl. + 46° 9′·8 (Pickering, and Copeland) (1900).

1911 Sept. 19th.—Readily found with the spectroscope, mag. 10·5, but before a proper examination could be obtained with high powers it clouded up.

Sept. 21st.—The nebula did not appear quite as sharply defined as a star, but definition not good. There was no star sufficiently near to connect by the micrometer.

Second Index Cat. 5117, R.A. $21^h\ 28^m\ 42^s$, Decl. + 44° 10′ (Fleming) (1900).

The spectroscope showed the nebula spectrum superimposed upon B.D. + 43°3962.

1911 Sept. 17th.—Called in *H.C.* 98 "exceeding faint," readily seen with the spectroscope. With powers up to 700 there seemed some nebulosity, but definition poor. Magnitude 10·5.

Sept. 21st.—Not very steady. A small 14 mag. star suspected between A and nebula, too faint to attempt measures. A distant coarse double star, mags. 12, 13·5, forms N. apex of a triangle, of which A and nebula are the base. The nebula differed little from a star in appearance.

Sept. 25th.—Not good definition, and could not see star between A and nebula. A faint star 14 mag. suspected at P 139° D 5″·7 from A. At times the nebula was thought to show a tiny disc.

Oct. 10th.—Thought to show a small disc.

The following are the results of micrometrical measures:—A is B.D. + 43°3962.

A and Nebula	1911·720	P 249°·0	D 23″·75	Mags. 9·2 10·2
	·731	247·1	23·76	9·5 10·5
	·772	245·6	24·00	
	·819	248·7	23·70	
	·821	249·3	23·75	
	·835	...	23·95	
Mean	1911·783	247·9	23·89	

Second Index Cat. No. 5217, R.A. $22^h\ 19^m\cdot9$, Decl. + 50° 28′ (Fleming) (1900).

1911 Sept. 1st.—Readily seen, magnitude 10·5. It lies N. of B.D. + 50°3700, and *p.* B.D. + 50° 3703. A distinct planetary nebula.

Sept. 6th.—Magnitude 10·5. Strong moonlight, planetary 2″·5, nucleus?

Sept. 7th.—Nebula 2″·5 ±, apparently elongated N. and S.

Sept. 25th.—Well seen on dark sky. Quite 4″, and apparently elongated N. and S., nucleus?

Oct. 11th.—Bright moonlight, well seen 3″ × 4″, longest axis N. and S.

Oct. 28th.—Quite 4″, nucleus?

The following places were obtained from B.D. + 50°3703:—

			m	s		′	″
1911·731	Δα	...	−0	17·13	Δδ	−0	29·75
·772	...	...		17·00	...	...	29·95
·821	...	...		17·16	...	...	29·80
·824	...	...		...	...	...	29·90
Mean 1911·787			−0	17·1		−0′	29·85

Standard Astronomy. By W. E. Cooke, M.A.

(1) Whatever may be the outcome of the present discussion on the above subject, there are two fundamental principles which ought to form the basis of any future scheme. These may be briefly enunciated thus:—

(*a*) The distinction between fundamental and differential observations must be regarded as radical.

(*b*) The differential character of differential observations should be preserved, and, if possible, shown in the printed catalogues.

(2) The former of these two is obvious now and becoming generally recognised. The failure to realise this in the past has been the cause of much misdirected energy, and even now seems to cause occasional misunderstanding. The principle itself lies at the root of the scheme adopted at the last Astrographic Conference. There is no necessity to dwell upon this point, but it is doubtful whether its full significance is even yet universally appreciated.

(3) The main object of this article is to advocate the principle briefly expressed in (*b*). I had originally intended to add this to my scheme placed before the Paris Conference, but was advised to let it drop for the moment, so as not to confuse the main points. I now desire to bring it forward for consideration by Fellows, because I think it is absolutely essential to any good scheme for the future.

(4) In a letter over my signature, published in the *Observatory* for January 1911, at the bottom of p. 48 is the following quotation from Boss's Preliminary General Catalogue:—

"It aims, also, to furnish a Standard Catalogue which shall include all stars likely to be needed at present for standards in zone observations and the like."

Against the principle underlying this I ventured to protest, but my letter also referred to other matters; consequently the particular point was lost, and, judging by the friendly criticism I received, both public and private, my attitude was, I think, misunderstood. May I be permitted to say briefly in defence, that my remarks *re* the relegation of Boss's (and all existing catalogues) to top shelves does not apply to those who conduct or discuss really fundamental results; and I probably attach as great importance as Professor Turner himself to the determination of proper motions.

(5) All the above is introductory. Permit me now to explain the principle which I wish to advocate as fundamentally essential to any new scheme, and which at present does not appear to be generally accepted.

Let A, B, C . . . represent catalogued positions of recognised fundamental stars; A_0 B_0 C_0 . . . the means of arbitrary groups of these, such as would be taken for determining clock error and

equator point on any observing night. Also let $a, b, c, \ldots$ represent positions of reference stars (not fundamental). The quantities we actually observe are $A_0 - a$, $B_0 - b$, etc.; and the principle I wish to advocate is that we should in some manner indicate these observed quantities in our differential catalogues.

(6) As far as I am aware, this has never yet been attempted except in the last two volumes (4 and 5) of the Perth catalogues. The usual procedure is to obtain the best available values for A_0, B_0, etc., and thus deduce $a, b, c, \ldots$ which are catalogued. That is all very well as far as it goes, but the quantities $A_0 - a$, etc., those actually observed, are, for any great scheme, the ones that we ought to preserve. Each of these quantities represents the actual difference at a definite epoch, subject *only* to errors of observation, between each reference star and a definite group of fundamentals. As time goes on the positions and proper motions of these latter stars will become known with greater and greater accuracy, hence also the positions of the reference stars at the epochs of observation.

(7) If our only object in differential observations were to obtain accurate present positions of a number of reference stars, I should still protest against Boss's suggestion, because I think we can obtain even more accurate places in the immediate future by the threefold scheme of the Paris Conference. But this is by the way, because we want much more than that. We desire, or ought to desire, to initiate a scheme whereby future astronomers can easily correct every individual observation in all branches of astrometry, so as to make it referable to the fundamental system then in use.

Proper motions, orbital motions, etc., will then stand out free from every source of error, except the accidental ones of observation. In a few decades we shall therefore have, not only extremely accurate positions, but very fair proper motions, for a great mass of stars distributed evenly throughout the sky, at the rate of 3 or 4 per square degree.

(8) The most important points in differential observations therefore are to determine the quantities $(A_0 - a)$, etc., with the greatest possible accuracy, and to show these differences in some practical form in the published results. In the long-run it is the *difference* that matters, not the immediately deduced star-co-ordinates.

(9) In order to observe these differences with the greatest accuracy, it is advisable to make the zone of observation as narrow as possible (say 10° for the intermediates and 2° for the reference stars). For ordinary zone work, therefore, we require in every 2° width of declination at least 4 intermediates for each hour of R.A., and these should be, for preference, stars of about the 8th magnitude. The *essentials* are—

(*a*) numerical sufficiency,
(*b*) proper distribution,
(*c*) suitable magnitude.

Boss's stars fail in each of these respects, and in lieu thereof give—

(*d*) accurate positions for a certain epoch,
(*e*) proper motions,

neither of which is in the least degree essential for the purpose.

(10) I hope Fellows will not take this as an attack upon Boss's catalogue, and thus again confuse the point at issue. I am quoting Boss's suggestions respecting one use for his catalogue, simply *because* it is the finest that has yet been issued. According to the old principles of zone-observing, his remarks are perfectly justifiable. But I am advocating the introduction of a new principle into any future scheme. At the back of everything will be a fundamental catalogue of comparatively few stars. My remarks have nothing whatever to do with the preparation of this. Let us take it for granted for the present purpose, and also let us assume that the absolute positions and proper motions of those stars will be determined with gradually increasing accuracy. The question I wish to ask is this:—

Can we not, immediately, so organise our work that from now onwards every astrometrical position may be referable, at any future time, to the latest edition of that fundamental catalogue?

Also, incidentally, will not the same organisation give us, as the first-fruits, the best available present positions for our observed places?

To both these questions I think an affirmative reply can be given, but only by recognising the importance of the *differences* actually observed in every piece of differential work, and preserving these differences somehow or other.

(11) Let us now take a glance at the broad scheme adopted by the last Astrographic Conference. The underlying principles will be the same whether the observations be made by transit circles, photography, or any other means.

At the back of everything comes the fundamental catalogue. Let us for the present purpose take this for granted, but note that it will probably contain those stars whose positions and proper motions have already been determined at least as accurately as any others. Even this, though advisable for immediate results, is not, however, essential. In the long-run *any* reasonably good set of positions will do for a start. The essential thing is that we have the accepted list as soon as possible. Also the accepted lists of the intermediates and reference stars. Never mind about accurate places just now, especially for these latter two. Our first step in differential work is to determine the *differences* between each of the intermediates and an arbitrary set of fundamentals, the $A_0 - a$, etc., already mentioned. Our next step is to determine the $A_0 - a$, etc., for the reference stars, where now A_0 refers to the intermediates used as standards. By means of these differences we refer first the intermediates and next the reference stars (and eventually all

astrometric observations, such as those of comets, planetoids, etc.) to any one of the successively published international catalogues we please, probably the latest and presumably most accurate. But in order to do this we must in some manner or other preserve the actually observed differences $A_0 - a$, and not smother them as we have done in the past. A practical method of carrying this into effect has been attempted in volumes 4 and 5 of the Perth Meridian Observations; but perhaps better methods will suggest themselves as the principle becomes more generally recognised.

(12) In conclusion, may I point out that this communication is concerned only with two points, viz.—

(*a*) The preservation in some form of the actual differences ($A_0 - a$, etc.) observed in differential observations in any future scheme of astrometry.

(*b*) And hence the necessity of choosing intermediate standards with special reference to number, distribution, and magnitude only, so that the errors of these differences may be reduced to the minimum. Any other principles are introduced briefly only for the purpose of illustrating the necessity for these two.

Perth Observatory, W.A.:
1911 *June* 26.

Addendum.—Those who wish to take up zone work on the reference stars under this scheme need not necessarily wait for the intermediates to be observed. It will, of course, be more convenient to start with good places for those standards, but this is by no means essential. As soon as the selection is made and approximate positions known the observations may be commenced. As long as the results are published in any manner analogous to the Perth vols. 4 and 5 they can easily be reduced at any time to the then existing system of intermediates or fundamentals, no matter what may be the errors of the standards now adopted. The only things we are waiting for are the three lists, or at least the two, viz. fundamental and intermediates; and as soon as these are available, differential observations under the new scheme may be immediately started.

Mesures d'étoiles doubles à l'Observatoire de Lille.
Par Robert Jonckheere.

Ces mesures font suite aux résultats déjà publiés dans le *Journal Astronomique.*

Dans la dernière colonne :

J = Jonckheere. V = Vanderdonck.

Nous avons le plaisir de joindre des mesures obtenues par le Prof. S. W. Burnham sur les étoiles J 58, J 112, J 117.

Étoiles.	Æ. 1920.	Dec.	Date. 1910+	P.	D.	Grandeurs.		n.	Obs.
	h m s	° ′		°	″				
Esp. 747	0 6 37	+50 8	1·644	179·7	3·08	9·1	10·0	1	J
			1·644	181·1	2·85	9·3	10·3	1	V
β 776	0 12 55	+50 8	1·644	194·6	0·98	8·8	9·0	1	J
			1·644	196·2	1·05	8·8	9·0	1	V
J 230	1 55 11	+24 41	1·847	153·0	2·34	9·1	9·1	1	J
			1·847	154·1	2·28	9·0	9·1	1	V
J 303	3 0 32	+ 7 46	1·847	36·7	2·24	8·9	9·5	1	J
			1·847	35·5	2·53	8·9	9·7	1	V
Ho. 504	39 25	+35 36	1·041	190·0	1·19	8·4	8·7	2	J
			1·041	191·2	1·24	8·2	8·5	2	V
J 306	3 53 9	+21 44	1·847	82·9	1·75	8·8	9·2	1	J
			1·847	82·1	1·80	8·9	9·3	1	V
H 677	4 21 15	+ 1 7	1·154	109·8	4·75	9·0	9·5	1	J
J 403	4 57 58	+ 2 39	1·847	204·9	2·02	8·9	9·8	1	J
			1·847	204·6	2·12	9·0	9·8	1	V
Ku. 21	5 2 29	+10 54	1·091	209·2	4·25	9·0	9·5	1	J
			1·091	209·5	4·38	9·1	9·5	1	V
Σ 668	9 41	− 8 18	1·172	201·8	9·46	...	8·0	2	J
			1·172	201·7	9·54	...	8·0	2	V
β 555	BC		1·172	202·9	0·19	8·2	8·5	2	J
Hn. 74	19 46	−17 16	1·138	234·4	5·65	9·2	9·2	1	J
J 7	26 16	− 2 52	0·987	205·4	1·40	8·7	9·2	1	J
			0·987	205·2	1·60	8·8	9·3	1	V
A 118	39 39	+13 17	1·790	355·8	2·83	9·1	9·3	1	J
J 50	58 21	+ 8 1	1·039	56·4	0·45	8·6	8·6	1	J
			1·039	56·2	0·47	8·6	8·6	1	V
J 51	5 59 2	+ 8 12	1·150	120·5	1·15	8·7	9·2	3	J
			1·150	119·5	1·18	9·0	9·3	3	V
Σ 840	6 2 1	+10 45	1·091	174·0	0·52	8·7	8·7	1	J

Étoiles.	Æ. 1920.	Dec.	Date. 1910+	P.	D.	Grandeurs.	n.	Obs.
	h m s	° ′		°	″			
Σ 850	6 3 25	− 3 59	0·833	16·4	2·17	8·8 9·8	1	J
			0·833	16·6	2·20	8·9 10·0	1	V
J 52	7 35	+ 2 29	1·159	281·5	1·97	8·3 9·2	2	J
			1·039	277·1	1·87	8·3 9·4	1	V
J 339	7 52	− 0 27	1·790	97·7	3·19	9·3 11·0	1	J
			1·790	98·8	3·00	9·4 11·0	1	V
Ho. 24	13 41	+ 9 21	1·773	152·3	3·54	8·2 11·0	1	J
			1·773	154·3	3·90	8·5 11·0	1	V
J 260	21 25	+ 7 47	1·161	177·8	1·33	8·0 9·6	2	J
			1·161	177·2	1·38	8·1 9·7	2	V
Ku. 25	22 53	+ 9 46	0·833	121·2	2·72	9·0 9·8	1	J
			0·833	122·3	2·70	9·0 9·7	1	V
Anderson 3	29 0	+ 5 0	0·926	196·2	12·80	6·5 11·8	1	J
Σ 938	30 26	+ 7 38	0·948	210·1	10·18	7·0 11·5	1	J
J 265	31 31	+ 5 12	1·869	220·9	4·39	9·4 10·0	1	J
J 412	38 37	+14 0	1·773	19·4	4·74	9·0 10·0	1	J
			1·773	18·3	4·67	9·2 9·6	1	V
Sirius	41 37	−16 36	1·099	87·1	8·75	... 9·8	4	J
			1·099	88·3	8·69	... 9·7	4	V
J 395	50 18	+13 55	1·773	159·5	1·95	9·0 9·0	1	J
Ho. 27	51 20	+20 12	1·773	127·8	3·22	8·9 9·1	1	J
			1·773	127·4	3·04	9·0 9·1	1	V
J 21	6 57 55	+10 38	1·216	273·1	2·94	9·1 9·4	2	J
			1·216	273·6	2·87	9·2 9·4	2	V
J 58	7 7 43	− 0 22	0·952	170·1	31·99	4·2 13·7	3	β
A 328	7 44	− 4 32	1·075	187·9	1·05	8·9 11·0	1	J
Roe. 25	10 52	+12 29	1·149	5·2	2·85	9·3 10·0	1	J
			1·149	4·6	2·82	9·2 9·8	1	V
OΣ 170	13 16	+ 9 27	1·195	107·6	1·56	7·0 7·0	1	J
			1·195	106·0	1·57	... ...	1	V
J 43	19 32	+ 8 25	1·205	68·2	2·94	9·4 9·7	2	J
Procyon	35 7	+ 5 26	1·069	28·6	4·69	... 9·4	2	J
Ho. 349	7 58 5	+12 40	1·091	225·1	9·64	6·9 11·9	1	J
			1·091	228·0	9·58	6·8 11·8	1	V
Σ 1196	8 7 38	+17 54	1·229	326·2	1·08	5·0 5·7	2	J
			1·229	325·6	1·09	5·0 5·5	2	V
	AC		1·229	109·5	5·16	5·0 5·7	2	J
			1·229	109·4	5·03	5·0 5·5	2	V
J 73	8 21 40	+ 8 7	1·139	209·4	3·54	8·0 10·5	2	J
			1·139	212·3	3·61	8·2 10·6	2	V

Étoiles.	Æ. 1920.	Dec.	Date. 1910+	P.	D.	Grandeurs.		n.	Obs.
	h m s	° ′		°	″				
J 77	9 0 45	+10 49	1·287	139·1	0″·71	8·6	8·6	2	J
			1·275	140·6	0·80	8·7	8·7	1	V
Σ 3121	13 10	+28 55	1·286	76·9	0·28	7·6	7·6	1	J
AG 166	22 39	+ 1 27	1·170	67·1	3·56	8·5	9·4	2	J
			1·170	68·1	3·72	8·5	9·5	2	V
Σ 1356	24 10	+ 9 24	1·283	122·4	1·06	6·5	7·0	2	J
			1·283	122·5	1·12	6·4	7·1	2	V
J 83	40 21	+ 3 59	1·298	50·9	1·63	9·1	9·2	2	J
			1·298	50·1	1·65	9·3	9·3	2	V
A. Clark 5	9 48 33	− 7 43	1·312	74·6	0·58	5·1	5·1	2	J
			1·301	75·2	0·62	5·2	5·2	2	V
OΣ 215	10 11 56	+18 9	1·244	202·2	1·06	7·0	7·0	2	J
			1·327	200·9	1·00	7·0	7·1	2	V
Σ 1424	15 34	+20 15	1·301	115·7	3·66	2·0	4·1	2	J
			1·295	115·8	3·86	2·0	4·0	3	V
J 84	32 3	+ 1 6	1·256	58·4	0·59	8·1	8·1	5	J
			1·273	58·2	0·63	8·1	8·1	4	V
J 79	35 7	+ 7 51	1·235	145·5	1·53	8·2	9·2	3	J
			1·235	146·3	1·53	8·3	9·2	3	V
J 89	39 20	− 3 27	1·273	86·1	3·77	9·0	9·9	2	J
			1·273	86·0	3·76	9·1	10·1	2	V
J 80	47 28	+ 7 0	1·235	217·1	4·01	8·7	9·3	3	J
			1·235	217·3	4·07	8·7	9·5	3	V
J 90	10 49 5	− 7 17	1·273	59·2	4·18	9·2	9·2	2	J
			1·273	59·5	4·20	9·2	9·2	2	V
	AC		1·273	191·3	11·54	...	12·0	2	J
			1·280	190·9	11·80	...	12·0	1	V
J 81	11 3 49	+10 38	1·222	140·5	1·89	8·8	8·9	2	J
			1·222	140·3	1·85	8·8	8·8	2	V
Σ 1523	13 55	+31 59	1·334	120·9	3·22	4·0	4·3	2	J
			1·317	121·8	3·09	4·0	4·3	3	V
Σ 1536	19 45	+10 56	1·336	40·9	2·06	4·0	7·0	3	J
			1·321	43·3	2·08	4·0	7·1	4	V
J 86	30 15	+ 4 34	1·304	94·6	1·81	8·6	9·6	3	J
			1·304	93·6	1·98	8·6	9·8	3	V
J 87	11 36 51	+ 5 24	1·343	127·7	1·60	7·5	11·0	1	J
			1·343	127·5	1·55	7·6	11·3	1	V
Σ 1670	12 37 36	− 1 1	1·332	146·5	6·17	3·0	3·1	2	J
			1·316	146·1	6·06	3·0	3·1	3	V
A 2233	16 22 48	+ 1 12	1·499	20·4	2·46	9·0	9·7	1	J

Étoiles.	Æ. 1920.	Dec.	Date. 1910+	P.	D.	Grandeurs.		n.	Obs.
	h m s	° ′		°	″				
A 1148	17 13 13	+ 7 31	1·409	353·3	1·54	8·8	9·9	2	J
			1·409	354·6	1·72	8·8	10·0	2	V
A.G. Berl. A 6241	19 18	+16 32	1·387	205·2	3·19	8·7	9·2	1	J
			1·387	203·8	3·30	8·6	9·3	1	V
A 2245	17 24 18	+ 2 16	1·488	346·4	1·98	8·5	9·4	1	J
			1·488	346·8	1·95	8·8	9·5	1	V
A 353	18 8 29	+ 4 14	1·488	11·6	2·14	8·6	11·0	1	J
			1·488	13·5	2·12	8·8	11·3	1	V
β 463	13 4	−16 54	1·401	91·3	1·71	8·7	9·2	1	J
			1·401	94·0	1·92	8·7	9·2	1	V
Σ 2310	17 17	+22 45	1·450	237·6	5·05	7·4	9·9	1	J
OΣ 347	20 53	+ 7 11	1·488	161·1	2·94	8·0	10·5	1	J
			1·488	167·5	3·18	7·8	10·4	1	V
J 95	24 3	+ 7 6	1·488	121·5	1·84	8·9	9·2	1	J
			1·488	119·9	1·91	9·0	9·4	1	V
J 96	29 6	+ 6 33	1·431	146·9	2·50	9·1	9·6	1	J
			1·431	146·6	2·41	9·1	9·8	1	V
Hu. 328	46 34	+20 38	1·524	186·3	4·55	8·9	9·7	1	J
J 112	18 58 51	+33 30	1·183	178·4	25·60	7·1	12·6	3	β
A 99	19 12 0	− 9 34	1·461	247·5	2·37	9·3	9·5	1	J
			1·461	245·6	2·15	9·3	9·4	1	V
J 117	24 53	+36 3	1·212	29·5	22·44	6·5	14·5	2	β
	AC		1·208	159·3	27·69	...	13·7	3	β
J 120	34 13	− 1 28	1·609	90·5	2·06	9·0	9·5	1	J
			1·609	88·6	2·40	9·0	9·5	1	V
A 1190	36 29	+12 0	1·412	198·4	4·95	8·9	9·5	1	J
J 151	49 2	+18 37	1·453	191·7	3·86	9·7	11·0	1	J
J 152	19 49 9	+ 7 29	1·795	167·0	3·01	9·0	10·5	1	J
Σ 2627	20 3 45	+ 4 32	1·792	32·5	1·71	9·3	10·0	1	J
			1·792	33·3	1·92	9·2	10·5	1	V
A 1199	6 9	+10 6	1·387	238·9	2·04	8·5	9·7	1	J
			1·387	242·4	2·11	8·6	10·3	1	V
A 1671	14 51	+13 2	1·387	234·7	1·74	9·2	9·6	1	J
			1·387	231·4	1·99	9·2	9·4	1	V
A 1680	33 24	+12 34	1·642	293·2	3·81	8·5	10·5	1	J
			1·642	295·5	3·46	8·5	11·0	1	V
β 267	37 33	− 4 41	1·551	237·1	2·17	9·2	9·2	1	J
			1·551	238·5	2·08	9·1	9·1	1	V
J 284	20 59 29	+16 34	1·636	231·8	2·57	9·5	12·0	1	J
Ho. 150	21 4 14	+18 31	1·847	137·2	2·33	8·8	10·0	1	J

Étoiles.	Æ. 1920.	Dec.	Date. 1910+	P.	D.	Grandeurs.		n.	Obs.
	h m s	° ′		°	″				
J 159	21 4 42	+ 9 20	1·776	244·5	4·05	8·8	11·0	1	J
			1·776	242·1	3·79	9·1	10·5	1	V
J 161	18 35	+10 37	1·740	123·7	1·80	9·0	9·2	2	J
			1·740	123·5	2·02	9·0	9·1	2	V
J 197	25 26	+14 30	1·741	25·0	2·91	9·1	9·2	2	J
			1·741	22·7	2·75	9·2	9·2	2	V
J 178	27 2	+11 20	1·855	184·8	2·25	9·4	9·7	1	J
			1·855	186·9	2·33	9·5	9·8	1	V
Hu. 278	37 44	+ 6 3	1·727	214·8	3·55	8·2	12·0	1	V
A 298	37 51	− 6 17	1·866	140·8	2·84	8·7	11·5	1	J
J 287	38 10	+15 55	1·866	350·0	1·94	8·9	9·5	1	J
			1·866	349·3	2·24	9·0	9·5	1	V
Hu. 279	38 16	+ 6 53	1·773	357·2	2·08	8·9	9·0	1	J
			1·773	356·0	2·18	9·0	9·1	1	V
J 288	21 49 48	+17 12	1·866	289·3	4·57	9·0	10·0	1	J
			1·866	289·8	4·66	9·1	10·4	1	V
J 179	22 15 21	+ 7 36	1·773	313·0	2·90	9·0	9·9	1	J
			1·773	314·5	3·18	9·1	10·0	1	V
J 292	30 33	+13 35	1·773	38·2	2·12	8·9	9·7	1	J
			1·773	40·6	2·42	9·0	10·0	1	V
J 165	33 56	+11 6	1·773	137·0	2·47	8·9	9·1	1	J
			1·773	136·6	2·54	9·0	9·4	1	V
J 210	46 33	+18 40	1·601	324·2	5·03	9·6	11·5	1	J
			1·601	322·4	5·10	9·3	10·5	1	V
H 1825	49 50	+13 10	1·776	218·9	1·79	9·0	9·2	1	J
			1·776	222·0	1·88	9·0	9·3	1	V
J 294	22 58 4	+10 0	1·773	137·7	4·24	9·2	9·2	1	J
			1·773	136·0	4·02	9·1	9·2	1	V
J 300	23 40 30	+15 49	1·773	7·2	3·57	8·9	9·8	1	J
			1·773	6·8	3·43	8·9	9·3	1	V
J 214	23 49 52	+15 32	1·773	110·6	3·42	9·0	9·8	1	J
			1·773	111·0	3·27	...	...	1	V

Notes.

Ho. 504.—Cette étoile est probablement en mouvement :

1896·97	185°·3	0″·75	2 η	Ho.
1901·83	187°·5	0″·90	5 η	Doo.

—A.G. Lund 1933.

Ku. 21.—Pas dans les A.G. B.D. + 10°714.

β 555.—L'allongement m'a semblé bien réel.

Hu. 74.—Pas dans les A.G. B.D. − 17°1109.

J. 7.—J'ai indentifié cette étoile pour B.D. − 2°1270. Les coordonnées ci dessus sont correctes.

Σ 840.—A.G. Lpz. I. 2006.

Σ 850.—A.G. Strass. 1923.

Ho. 24.—A.G. Lpz. II. 2779.

Ku. 25.—Pas dans les A.G.

Anderson 3.—A.G. Lpz. II. 2988. Ce catalogue donne la grandeur 8·3.

Σ 938.—Lpz. II. 3037.

J. 58 β.—The proper motion of the large star is small; Auwers 0″·034 in 322°·1. Boss 0″·011 in 344°·8.

Procyon.—Mesures très difficiles.

Ho. 349.—A.G. Lpz. I. 3214.

A.G. Berl. A. 6241.—Les A.G. donnent "Com. 9·3m, 6″ 200°."

A. Clark 5.—A.G. Vien-Otta 3832.

J. 112 β.—D.M. (33°) 3295 wrongly identified by J. as the 6·8m star, D.M. (33°) 3287. The place of the star measured for 1900 is : R.A. $18^h\ 58^m\ 6^s$, Decl. + 33° 48′·6. (Il faut probablement lire + 33° 28′·6. J. 112 est donc A.G. Leiden 7050. La position donnée ici est exacte.—J.)

A. 99.—Aitken donne 67°·1, 1″·96, 10·0, 10·0, 1900·49.

J. 151.—Cette étoile n'est pas A.G. Berl. A. 7756 mais se trouve juste à côté.

A. 1671.—Aitken donne 46°·7, 1″·98, 9·5, 9·5, 1907·64.

H. 1825.—A.G. Lpz. I. 9125.

Observatoire de Lille,
25 Novembre 1911.

Nouvelles étoiles doubles. 7ème Série.

Par Robert Jonckheere.

La distance angulaire moyenne, pour la totalité des couples publiés, est de 2″·8 pour les étoiles plus faibles que la 7ème grandeur.

Dans la dernière colonne :

J = Jonckheere. V = Vanderdonck.

J.	Catalogue.	Æ. 1920. h	m	s	Dec. °	′	Date. 1911	P. °	D. ″	Grandeurs.		n.	Obs.
601	+10° 4093	19	51	44	+10	22	·801	252·4	4·11	9·0	9·7	1	J
							·801	251·6	3·99	9·2	9·8	1	V
602	Anon.	20	0	35	+10	23	·795	330·6	4·20	9·2	12·0	1	J
							·795	329·6	4·08	9·3	11·8	1	V
603	+ 4° 4339		1	41	+ 4	20	·792	126·5	2·88	9·3	9·4	1	J
							·792	125·9	2·65	9·4	9·5	1	V
604	+11° 4188		11	18	+11	18	·795	231·4	0·53	8·9	9·1	1	J
							·795	230·0	0·55	8·8	9·1	1	V
605	Anon.		49	2	+17	10	·858	284·3	3·20	9·5	10·0	1	J
							·858	287·1	2·85	9·5	10·0	1	V
606	Anon.		52	13	+20	8	·817	184·4	2·64	9·5	9·7	1	J
							·817	184·0	2·43	9·7	9·9	1	V
607	+17° 4267		53	25	+17	42	·839	295·9	2·24	9·4	11·0	1	J
608	Anon.	20	55	37	+10	54	·795	99·4	4·84	9·2	11·5	1	J
609	Anon.	21	9	30	+14	43	·798	250·0	3·19	9·4	11·5	1	J
610	Anon.		11	32	+43	9	·831	22·5	3·29	9·5	10·7	1	J
							·831	22·8	2·95	9·5	10·8	1	V
611	Anon.		25	3	+20	7	·836	21·6	1·31	9·5	9·5	2	J
							·839	19·1	1·20	9·5	9·5	1	V
612	Berl. B 8251		26	17	+23	32	·817	274·0	1·94	8·9	10·0	1	J
							·817	275·1	1·70	9·0	10·2	1	V
613	Anon.		29	43	+19	56	·847	209·7	4·31	9·0	12·5	1	J
614	Anon.		39	2	+20	41	·839	297·9	1·67	9·1	9·6	1	J
							·839	300·7	1·45	9·3	9·8	1	V
615	− 2° 5634		42	30	− 1	43	·866	347·7	4·21	9·4	10·6	1	J
							·866	351·1	3·99	9·5	11·2	1	V
616	Anon.	21	43	35	+45	7	·831	311·4	4·73	9·5	10·5	1	J
							·831	318·4	4·33	9·3	11·0	1	V
617	+21° 4781	22	29	47	+21	52	·833	283·0	0·82	8·8	9·1	1	J
618	Anon.	22	36	44	+16	9	·795	41·6	1·94	9·5	9·5	1	J
							·795	44·0	1·86	9·5	9·5	1	V

J.	Catalogue.	Æ. 1920. h m s	Dec. ° ′	Date. 1911	P. °	D. ″	Grandeurs.		n.	Obs.
619	Anon.	22 50 00	+17 55	·795	221·2	2″14	9·3	11·0	1	J
620	Anon.	50 39	+17 52	·795	77·9	1·92	9·8	10·5	1	J
621	Anon.	22 50 47	+17 6	·795	44·8	2·92	9·5	9·8	1	J
622	Anon.	23 0 53	+ 0 57	·790	311·8	3·63	9·2	9·8	1	J
				·790	313·6	3·52	9·3	10·0	1	V
623	Anon.	8 3	+35 59	·869	234·5	1·87	9·6	9·8	1	J
				·869	227·8	1·57	9·5	9·7	1	V
624	Anon.	10 55	+24 21	·855	9·8	2·59	9·3	9·4	1	J
				·855	8·5	2·74	9·4	9·5	1	V
625	Anon.	33 8	+18 51	·795	256·0	2·44	9·5	11·8	1	J
				·795	257·5	2·39	9·4	12·0	1	V
626	+29° 4990	42 4	+30 3	·831	233·8	4·95	9·0	10·0	1	J
				·831	233·1	5·32	9·0	10·6	1	V
627	Anon.	48 3	+17 35	·855	182·9	2·42	9·5	11·0	1	J
628	Anon.	23 59 53	+21 4	·795	195·6	3·18	9·8	9·8	1	J
				·795	195·8	2·92	9·8	9·8	1	V
629	Anon.	0 0 56	+20 58	·795	295·2	0·80	9·5	9·7	1	J
				·795	294·1	0·91	9·5	9·8	1	V
630	+20° 26	16 1	+21 3	·855	108·4	1·89	9·1	9·8	1	J
				·855	106·6	2·26	9·1	9·8	1	V
631	Anon.	21 36	+ 3 47	·790	111·9	3·74	9·6	9·7	1	J
				·790	110·5	3·37	9·5	9·8	1	V
632	+ 3° 42	21 51	+ 3 40	·790	78·3	1·50	9·3	9·4	1	J
				·790	74·1	1·41	9·2	9 5	1	V
633	+ 3° 52	25 48	+ 4 19	·790	317·2	1·91	9·5	11·5	1	J
634	Anon.	26 43	+21 8	·855	278·3	1·82	9·7	11·0	1	J
635	+20° 86	35 35	+20 53	·795	133·5	3·35	9·1	11·0	1	J
				·795	131·8	2·92	9·1	11·2	1	V
636	+22° 125	43 40	+22 38	·792	260·5	1·31	9·3	9·8	1	J
				·792	260·6	1·35	9·2	9·8	1	V
637	+ 7° 129	49 50	+ 8 10	·866	169·2	2·62	8·9	9·4	1	J
				·866	172·9	2·32	9·0	9·6	1	V
638	Anon.	0 50 52	+ 8 7	·866	199·4	2·87	9·5	9·5	1	J
				·866	205·2	2·46	9·5	9·5	1	V
639	Anon.	1 20 12	+24 24	·795	119·4	3·73	9·6	9·6	1	J
				·795	123·8	3·98	9·5	9·5	1	V
640	+ 4° 268	28 50	+ 5 18	·790	47·4	1·54	9·2	9·2	1	J
				·790	47·1	1·54	9·1	9·1	1	V
641	Anon.	1 43 45	+ 9 20	·866	244·6	4·97	9·0	10·0	1	J
				·866	245·5	5·27	8·9	10·5	1	V

J.	Catalogue.	Æ. 1920. h m s	Dec. ° ′	Date. 1911	P. °	D. ″	Grandeurs.		n.	Obs.
642	+26° 298	1 43 53	+26 49	·792	112·8	1·20	9·0	10·0	1	J
				·792	115·0	0·99	9·0	11·0	1	V
643	+13° 304	53 3	+14 8	·869	164·0	3·74	8·9	10·0	1	J
				·869	167·1	4·32	8·9	10·0	1	V
644	Anon.	54 56	+ 7 11	·869	209·2	1·53	9·8	11·5	1	J
645	Anon.	55 18	+52 1	·869	195·5	2·71	9·6	9·8	1	J
				·869	195·8	2·97	9·4	9·6	1	V
646	Anon.	1 55 55	+27 19	·795	79·8	4·63	9·1	10·5	1	J
647	+26° 414	2 23 58	+26 27	·855	133·9	3·63	9·0	12·5	1	J
648	+ 0° 406	24 48	+ 0 23	·847	154·7	2·59	9·1	9·4	1	J
				·847	159·1	2·56	9·3	9·6	1	V
649	Anon.	2 40 11	+49 32	·869	125·2	4·35	9·4	9·8	1	J
				·869	126·5	4·25	9·4	10·2	1	V
650	Anon.	4 45 15	+48 53	·872	33·7	4·11	9·0	9·9	1	J
				·872	33·7	4·40	9·1	10·2	1	V
651	+48° 1200	4 55 33	+48 58	·872	233·2	3·95	8·9	9·9	1	J
				·872	232·2	4·13	9·0	9·9	1	V
652	Anon.	5 15 27	+33 45	·869	356·2	3·83	9·4	9·5	1	J
				·869	353·2	3·72	9·4	9·6	1	V
653	Anon.	29 46	+35 14	·869	169·3	5·03	9·5	11·0	1	J
				·869	171·9	5·33	9·4	11·2	1	V
654	Anon.	29 47	+12 45	·869	250·0	4·00	9·2	9·4	1	J
				·869	250·5	4·28	9·3	9·6	1	V
655	+14° 1015	40 40	+14 8	·869	171·7	2·21	9·0	11·0	1	J
				·869	172·1	2·40	9·1	11·0	1	V
656	Lpz. I. 1858	48 40	+14 23	·869	99·7	3·70	9·1	10·0	1	J
				·869	101·3	3·94	9·1	10·3	1	V
657	Anon.	5 51 49	+12 14	·866	213·3	4·25	9·6	10·0	1	J
				·866	214·8	4·06	9·5	9·9	1	V
658	Anon.	6 23 57	+ 5 59	·869	271·4	2·70	9·8	10·0	1	J
				·869	274·7	2·72	9·7	9·9	1	V
659	+ 4° 1269	24 26	+ 4 56	·869	232·1	2·40	9·2	11·0	1	J
				·869	235·4	2·69	9·1	11·0	1	V
660	+ 4° 1270	24 26	+ 4 52	·869	87·4	3·00	9·0	9·1	1	J
				·869	93·3	2·28	9·1	9·1	1	V
661	Anon.	28 3	+ 5 46	·869	141·6	3·70	9·4	9·4	1	J
				·869	139·0	3·81	9·4	9·5	1	V
662	Anon.	6 28 29	+ 6 3	·869	101·6	3·97	9·0	11·0	1	J
				·869	102·2	4·11	9·2	11·0	1	V

J.	Catalogue.	Æ. 1920.	Dec.	Date.	P.	D.	Grandeurs.		n.	Obs.
		h m s	° ′	1911	°	″				
663	+12° 1189	6 28 34	+12 32	·790	119·4	4·53	9·1	9·7	1	J
				·790	119·6	4·64	9·2	9·8	1	V
664	Anon.	30 20	+ 5 39	·869	179·8	3·23	10·0	11·0	1	J
				·869	181·6	3·29	9·8	10·7	1	V
665	+38° 1585	38 38	+38 27	·866	126·0	3·65	9·1	9·3	1	J
666	Lpz. II. 3177	39 37	+ 7 34	·869	25·7	2·09	8·0	11·5	1	J
				·869	26·8	1·90	8·3	11·7	1	V
	AC			·869	22·1	14·14	8·0	10·5	1	J
				·869	23·9	14·35	8·3	10·7	1	V
667	Lpz. II. 3181	6 39 57	+ 7 30	·869	71·0	4·25	8·7	10·0	1	J
				·869	72·2	4·00	8·9	10·0	1	V

604. Le B. D. ne donne que la grandeur 9·3. Cette étoile est cependant beaucoup plus brillante que B. D. 11° 4190 indiquée 9·3 également.

613. D'autres étoiles voisines plus faibles, figurent dans le B. D.

621. Aussi une étoile de 11ème grandeur à 90°, 15″ ±.

649. Le B. D. donne la grandeur 8·4.

Erratum.—M.N., vol. LXXI., No. 9, page 753, deuxième mesure de J 483, pour 22″·3 lire 2″·23.

Observatoire de Lille,
le 20 Novembre 1911.

Prevention of Dew-deposit on Glass Surfaces.

By J. Franklin-Adams.

In May 1910 I sent to the *Monthly Notices* a description of an apparatus for the prevention of dew-deposit.

In accordance with requests from readers of the above note, a detailed drawing and further description are now given.

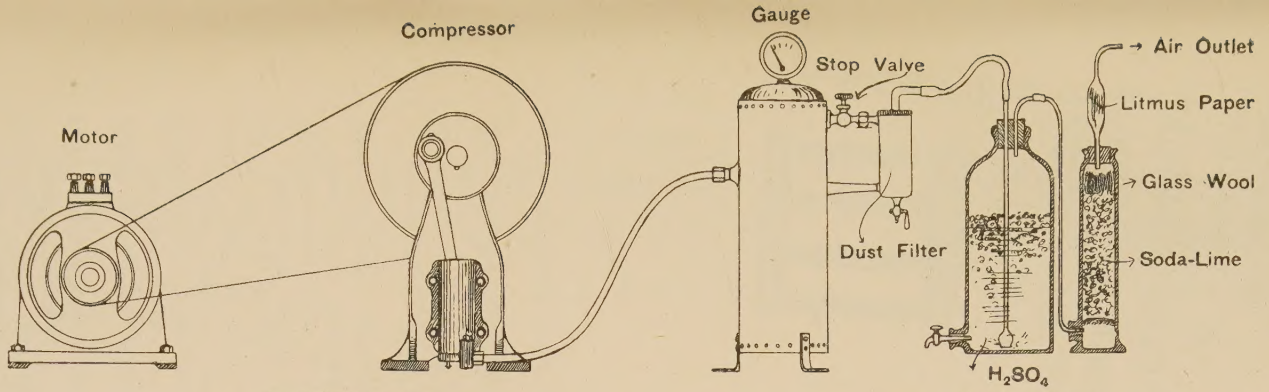
The Mervel Hill photography of the northern hemisphere, consisting of 420 plates fifteen inches square, might have been completed in two years instead of in six if the glass surfaces could have been kept free from dew, but no record could be found, even after much search and inquiry, of any experiments to help towards such an apparatus.

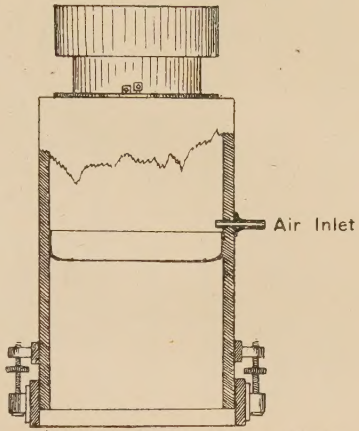
Oxygen from iron bottles gave the lines of research, and soon a system, amply illustrated by the enclosed diagram, was evolved.

The air is first passed through a glass reservoir containing pumice-stone steeped in sulphuric acid, and this with us has been successful.

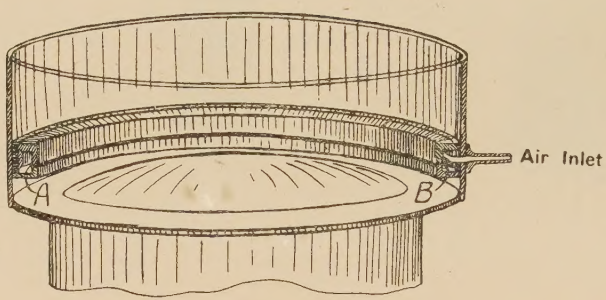
For extremely delicate work, and to remove any trace of moisture, a tube or bulb containing phosphorus pentoxide can further be added.

The air then passes into a tube containing soda lime to remove any acid that may have been carried over; after passing through a plug of glass wool it impinges upon a piece of litmus paper, whence it is conveyed in india-rubber tubes into an air-space in a dew cap made of two tubes; from this it escapes through holes or slits, impinging at a low velocity upon the upper surface of the upper lens. Another tube conveys dry air into the body of the camera, whence, after drying the other lens surfaces and the sensitive side of the photograph plate, it escapes through interstices into the outer air.





Air Inlet to Dry Plate and Inner Surface of Objective



Air Distributing-Slit around Outer Surface